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PRICE PREMIA FOR CHEAPEST-TO-DELIVER BONDS

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Abstract

This paper tests for price pressure effects in the cash U.S. Treasury market for bonds that are the cheapest-to-deliver (CTD) instrument in the Treasury bond futures market. We find that there is a detectable and significant premium in the cash market of roughly 14 cents, on average, for deliverable bonds that are CTD, which is both statistically and economically significant. Further, we find that CTD status remains a significant factor in the relative pricing of these securities after accounting for control variables. Our results add to the body of literature examining idiosyncratic price behavior within the U.S. Treasury market.

Keywords

Treasury bond, Bond futures, Cheapest-to-deliver, Relative value

JEL classification: G12, G13

1. Introduction

The impact that trading activity in derivative securities can have on prices in the underlying, primary asset market is a central research and public policy issue. While a number of researchers beginning with Figlewski (1981) have explored how trading in derivatives affects the volatility of the underlying security and its market, less attention has been directed to potential distortions in the pricing of the primary asset itself. In two well-known case studies, Jordan and Kuipers (1997) and Merrick, Naik and Yadav (2005) provide some direct evidence on this subject by tracing price disruptions in the cash US Treasury and British government bond markets, respectively, to trading activity in their associated bond futures markets. In these two cases at least, they show that derivatives market activity can influence the price of the corresponding primary market asset to an extreme degree.

In this study, we examine the broader possibility that the cheapest-to-deliver (CTD) bond in the Treasury futures market typically exhibits a detectable price premium in the cash Treasury bond market. This issue is important to policy makers, researchers, and practitioners for two reasons. First, in light of monetary policy needs along with transient budget surpluses and deficits, both the U.S. Treasury and the Federal Reserve in recent years have systematically engaged in the buyback and open market purchases and sales of Treasury securities. The securities involved often are concentrated in long-term Treasury bonds of deliverable grade in the bond futures market. If the subsequent impact on the supply of such bonds leads to inadvertent surpluses or shortages in the cash market when these bonds become CTD, these initiatives can lead to unanticipated market instability (D'Amico and King, 2013).

Second, a question of continuing interest for researchers and traders alike concerns the sources of relative price differences for U.S. Treasury securities. While tax and liquidity effects have been documented in the Treasury market by a number of researchers, the effect of futures trading has not been directly addressed, at least in a general fashion as studied in this paper. If futures trading leads to observed price effects for CTD bonds, it is of interest to know the magnitude and significance of this effect compared to other market frictions. Any potential price pressure effects have direct implications for relative value trades in the Treasury market, and indirectly impact efforts to measure the term structure of interest rates with maximum efficiency.[i]

In an effort to provide empirical evidence on this issue, we follow the extant literature on relative value in the Treasury securities market by examining price effects due to factors outside the term structure alone. The specific instruments of interest in this study are noncallable U.S. Treasury bonds deliverable into either the lead or first-deferred Treasury bond futures contract trading at the Chicago Board of Trade (CBT) during our sample period.[ii] Using several different estimation techniques to mitigate any method-specific results, we calculate the magnitude and sign of daily price differences between cash market bonds and synthetic bonds of identical coupon and maturity over the period 1990 through 1994. The resulting price difference is examined in pooled time-series, cross-sectional regression analysis to determine whether market frictions, including CTD status in the futures market, can explain the variability in the estimated bond pricing errors over time.

We find that there is a detectable and significant price effect for bonds in the cash Treasury market when they are CTD in the Treasury bond futures market. Specifically, we document a price pressure effect of 14 cents, on average, for bonds with $100 face value; we find this premium to be both statistically and economically significant. Further, we show that this futures-related pricing factor for cash market Treasury bonds is robust to the presence of market frictions, such as tax and liquidity effects, studied in previous research. Our study provides context for the case studies examined in extant research, by documenting a general, albeit less pronounced, price pressure effect between these two markets. We conclude that our findings suggest this observable characteristic of deliverable instruments should be included as a control variable in future research.

The remainder of the paper is organized as follows. The next section describes the cheapest to deliver quality option associated with CBT Treasury bond futures and outlines the research design for this study. Section 3 describes the data used in the paper, and Section 4 discusses our empirical results. We conclude the paper with some final remarks in Section 5.

2. Background

In the absence of market frictions, the price of a U.S. government security, by virtue of its lack of credit risk, would be determined solely by the term structure of interest rates. For fixed-coupon noncallable U.S. Treasury bonds, this implies that given a cash flow payment schedule, the bond price is simply the vector product of the discount function and the bond's payment stream. However, even given observation noise, it is well-known that the price of U.S. Treasury bonds are not completely determined by the term structure. Rather, bond pricing errors have been shown to be systematically related to various bond characteristics and market frictions, including relative coupon, tax, and liquidity effects.

The goal of this paper is to examine whether an additional pricing factor for U.S. Treasury bonds exists due to their value as the CTD instrument in the Treasury bond futures market. Before we test for such an effect, we first review the source of the CTD quality option.

2.1. CBT Treasury bond futures and the cheapest to deliver option.

The U.S. Treasury bond futures contract traded at the Chicago Board of Trade (CBT) has been one of the most successful financial derivatives products ever offered. Since its introduction in 1977, trading in CBT bond futures has grown to an average daily volume on the order of 7 million contracts, or $700 billion in face value of underlying bonds. At times, the activity has been much higher. An interesting feature of the CBT bond futures (and many other exchange-traded futures contracts) is the quality option embedded in the contract. This option stems from the right of the short party to make delivery of any of a qualified set of securities during a fixed interval of time (Chance and Hemler, 1993).

For CBT bond futures, a U.S. Treasury bond is eligible for delivery if, on the first calendar day of the contract expiration month, the bond will have at least 15 years remaining maturity (or 15 years to the first call date if the Treasury bond is callable). However, since the individual securities in the deliverable set are heterogeneous with respect to coupon, maturity, and other characteristics, the exchange must devise a system to put each deliverable bond on an approximately equal footing. For CBT bond futures, the method used to equalize delivery-grade bonds is known as the conversion factor.

If the yield curve is flat at a level of 6%, then all bonds will be on an equal footing in the CBT conversion factor system.[iii] When the yield curve is not flat at 6%, some bonds will be less costly to deliver than others.[iv] If any relative pricing idiosyncracies exist amongst deliverable bonds, the cost of delivery will be further impacted, potentially making some bonds very cheap to deliver and others correspondingly dear. Naturally, since the short investor controls the delivery option, the cheapest bond is the one that will ultimately be delivered (if delivery is made). Accordingly, the futures price closely tracks price movements in the so-called cheapest-to-deliver (CTD) bond, and market participants are constantly aware of which bond is CTD against the futures contract at any one time.

To actually identify the CTD bond, three pieces of information are necessary: the price of the deliverable bond under consideration, the current futures price, and the conversion factor for the bond. The specific procedure used, which involves calculating an implied repo rate (IRR), is reviewed in Appendix A.

2.2. Price slack in the bond futures market.

The most direct method for testing whether CTD bonds exhibit significantly different price behavior than non-CTD bonds is to compare the cash market prices for each to that of otherwise identical, generic bonds; i.e., bonds with the same coupon rates and maturities. We would expect that, on average, CTD bonds will be relatively underpriced compared to non-CTD bonds—almost by definition. For example, if a particular deliverable bond is grossly underpriced in the cash market, for whatever reason, then it will almost surely be the CTD bond in the futures market.

A more interesting approach is to explore the relative value of CTD bonds from the perspective of the futures market. As discussed in Appendix A, the CTD instrument for a given futures contract is the issue with the largest IRR on a given day. The greater the difference between the IRR for the CTD bond and the next-cheapest-to-deliver bond, the cheaper the CTD bond is from the perspective of the futures market. Hence, there is a certain amount of "slack" in the cash market price of the CTD bond from the perspective of futures market traders; i.e., the CTD bond price can increase, to a degree, until a different bond takes its place as the CTD instrument. If one calculates how large the allowable price increase until a substitution takes place, a measure of the relative degree of cheapness exhibited by CTD bonds compared to non-CTD deliverable instruments results, referred to as the "price slack" in the futures contract in Jordan and Kuipers (1997); this calculation is reviewed in Appendix B.

However, even this approach may not provide a complete picture of the potential impact on the cash market price of CTD bonds as a result of bond futures market demand and trading activity. In effect, the price slack discussed above may understate the true amount of slack, at least at certain times. The reason is that the slack is calculated using the observed cash market price of the CTD bond. If the bond is bid up in the cash market by futures traders when the slack is large, the true slack will be obscured, since one only observes the cash bond's price *ex post*. To the futures market, this price increase does not substantially change anything, since the bond remains CTD in either case (albeit not as cheap as it once was). But from the perspective of the cash market, the result of this price pressure effect is more meaningful, as it can conceivably cause a bond that was once cheap relative to other cash market securities to become relatively dear instead.

To quantify this effect, we re-calculate slack as described in Appendix B, but use the price of an otherwise identical deliverable bond's price in the calculation rather than the observed cash market price of the CTD bond itself. Of course, such an ideal substitute bond does not generally exist, so we synthesize it from a cash-flow replicating portfolio of U.S. Treasury coupon STRIPS.[v] For purposes of the calculation, we use the lower of the actual cash market price of the CTD bond and the STRIPS-synthesized price, so that the slack so constructed can be interpreted as an upper bound on the magnitude of slack that exists in the bond futures contract. This slack variable may provide a more informative view of the impact of CTD futures market status on cash market deliverable bond prices.

Table 1 Daily price slack in the CBT Treasury bond futures contract, 1990–1994.

The table reports the following summary statistics on the potential price slack in the lead and first deferred CBT Treasury bond futures contract, using daily settlement prices, over the 1,250 trading days covering the period 1990–1994: the mean, standard deviation, median, minimum, maximum, and interquartile range, as well as the difference in mean price slack across contracts and CTD bond type.

Heteroskedasticity and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987) are used in constructing the *t*-statistics for testing the null hypothesis that the difference in group means is zero. Potential price slack is calculated as the allowable increase in the synthetic price of the cheapest-to-deliver (CTD) bond such that the bond is no longer CTD, and is measured in units of dollars assuming a $100 face value bond (see Appendix B for details).

Lead Statistic	Contract		Deferred Contract
N	1,250		1,250
Mean	0.2038		0.2119
Difference in means		–0.0081	
t-statistic		2.91	
Standard deviation	0.2687		0.2627
Minimum	0.0002		0
Maximum	1.8534		1.8378
Median	0.1305		0.1428
Interquartile range	0.1863		0.1931

Table 1 provides some descriptive statistics on the slack variable described above, calculated for both the lead and first-deferred CBT bond futures contract, over the 1,250 trading days covering the sample period 1990–1994.[vi] As shown, the average slack value for both contracts exceeds 20 cents, with the median somewhat smaller at 13 to 14 cents. Half of the observations occur within a range of roughly 19 cents, and slack for the lead contract is less than one cent lower than for the deferred, which is statistically, though not economically, significant.[vii]

Based on this preliminary evidence, the data in Table 1 suggests the slack variable represents an opportunity to test for relative value in the cash Treasury market due to CTD status in the Treasury futures market. We pursue this further in the presentation of our detailed empirical results in Section 4.

2.3. Research design.

To determine whether there is a detectable impact on the cash market price of noncallable deliverable bonds that are CTD in the futures market, we need an estimate of a deliverable bond's price with the same coupon rate and maturity, but whose price is determined solely by the term structure of interest rates. Since such bonds generally do not exist, we synthesize them using alternate estimation techniques, in an effort to mitigate any method-specific results. A similar approach is used throughout the Treasury richness/cheapness literature. Further, we need to use methods for pricing these bonds independent of the STRIPS replicating portfolio approach used for constructing the slack variable discussed earlier.

We follow Jegadeesh (1993), Jordan and Jordan (1997) and others in constructing synthetic bond prices for our study by employing three alternate methods fit to the daily noncallable coupon Treasury note and bond yield curve: (1) linear interpolation;[viii] (2) the one-factor Cox, Ingersoll and Ross (CIR, 1985) term structure model; and (3), traditional cubic splines in the manner of Litzenberger and Rolfo (1984b). For all three methods, the fitted yield curves are used to calculate a synthetic coupon bond price via simple present value calculations.

Using any of these three techniques, we subsequently examine the pricing errors between the synthesized and actual prices for noncallable deliverable Treasury bonds in pooled time-series, cross-sectional regression analysis, similar to the approach used elsewhere in this literature (e.g., Elton and Green, 1998). Our focus in this study is to explore whether CTD status is a significant determinant of the relative pricing of deliverable bonds. We include slack as an explanatory variable in the analysis, and also test for robustness by including proxy variables for market frictions such as liquidity and tax effects.

3. Data description

Three different coincident data sources are necessary for the research design of this study.[ix] The first set of data consist of over 150,000 daily bid and ask yield quotations on U.S. Treasury coupon STRIPS over the 1,250 trading days in the period January 1990 through December 1994. The raw data is identical to the quotes that underlie data reported in *The Wall Street Journal* during this time period. The data is adjusted to reflect actual market practice as described in Jordan and Kuipers (2005).

The second set of data consist of nearly 25,000 daily price quotations on cash market, noncallable, deliverable U.S. Treasury bonds during the same 1990–1994 sample period. This data is a subset of a complete record of daily price quotes on Treasury notes, bonds and bills obtained from the Federal Reserve Bank of New York (FRBNY). For this study, we discard observations for all instruments on day *t* that are not deliverable against the lead or first-deferred CBT Treasury bond futures contract. Because most of our calculations require the full (with interest) price of coupon-bearing Treasury securities, we must calculate accrued interest for deliverable cash bonds from the supplied price quotes. We use next-day settlement in these calculations, consistent with market practice and previous research.

The final set of data consist of daily settlement prices for the U.S. Treasury bond futures contracts traded at the Chicago Board of Trade (CBT) over the 1990–1994 sample period. The futures prices are from Tick Data, Inc., a data vendor specializing in futures and options markets. We also obtained the CBT's quarterly *Market Summary* publications, which reports the delivery history of bonds for expired futures contracts and the conversion factors for outstanding bonds against current contracts, over the five-year sample period. We reviewed the *Market Summary* reports as a cross-check on the conversion factors we calculate for deliverable bonds.

4. Results

4.1. Regression tests for cash bond pricing: CTD status and price slack.

For each of the 24,380 daily observations on noncallable U.S. Treasury bonds deliverable against the lead or first-deferred CBT Treasury bond futures contract over the period 1990–1994, we test for pricing effects in the cash bond market due to CTD status in the futures market by estimating the following pooled time series and cross-sectional regression:

$$\left(P^{cash}_{actual} - P^{cash}_{predicted}\right)_{j,t} = \beta_0 + \beta_1 \cdot CTD_{j,t} + \beta_2 \cdot CTD_{j,t} \cdot SLACK_{j,t} + \varepsilon_{j,t} \quad . \qquad (1)$$

The dependent variable in Eq. (1) is alternately the bond price prediction error constructed from daily spline fits of the Treasury yield curve (*DPSPL*), daily fits for the Cox, Ingersoll and Ross (CIR, 1985) term structure model (*DPCIR*), and linear interpolation along the daily noncallable Treasury yield curve (*DPLIY*). The independent variable $CTD_{j,t}$ is a dummy variable equal to one if bond observation *j* on day *t* is the CTD bond for either the lead or deferred futures contract (or both) and zero otherwise; *SLACK* is the potential price pressure effect variable discussed in Section 2; and $\varepsilon_{j,t}$ is the regression residual. Table 2 presents the results from estimating Eq. (1) for each of the three methods of generating synthetic prices.

Table 2 Coefficient estimates from regressions of daily estimated pricing errors for noncallable U.S. Treasury bonds deliverable against the lead or first deferred CBT Treasury bond futures contract on futures-related variables, 1990–1994.

The dependent variable is the daily pricing error for 24,380 observations on noncallable U.S. Treasury bonds deliverable against the lead or first deferred CBT Treasury bond futures contract over the period 1990–1994. The pricing error is calculated as the observed full (with accrued interest) price of the bond minus its synthetic price, where the synthetic price is estimated using fits to the daily U.S. Treasury yield curve from three methods: cubic splines (*DPSPL*), the one-factor Cox, Ingersoll and Ross (1985) term structure model (*DPCIR*), and linear interpolation (*DPLIY*). The pricing errors are measured in units of dollars assuming a $100 face value bond. Heteroskedasticity and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987) are used in constructing the *t*-statistics (shown in parentheses).

Pricing Method	Intercept	*CTD*[a]	*CTD•SLACK*[b]	adj. R^2
DPSPL	0.1463	–0.0506		0.002
	-45.98	*-7.02*		
	0.1463	–0.1918	0.9316	0.015
	-45.98	*-15.97*	*-11.58*	
DPCIR	0.2352	–0.2050		0.012
	-38.31	*-20.87*		
	0.2352	–0.3349	0.8573	0.016
	-38.31	*-24.29*	*-12.36*	
DPLIY	0.0171	–0.0797		0.009
	-6.14	*-17.15*		
	0.0171	–0.0554	0.1602	0.01
	-6.14	*-7.56*	*-3.16*	

[a] Dummy variable CTD is equal to one if the bond observation is cheapest to deliver against the lead or first deferred CBT Treasury bond futures contract (or both), else zero.

[b] Price slack in the Treasury bond futures contract on day *t*, calculated as the allowable increase in the synthetic price of the CTD bond such that the bond would no longer be CTD, measured in units of dollars assuming a $100 face value bond (see Appendix B for details). For bond observations CTD against only the first-deferred contract, the slack for that contract is used in the regression analysis. For bond observations CTD against the lead contract or both contracts, the slack used in the regression is for the lead contract.

Examining the first row of results for *DPSPL*, *DPCIR*, and *DPLIY*, the evidence shows that on average, CTD bonds are cheaper than non-CTD bonds, as one would expect given their cheap delivery status. Depending on the method used to construct synthetic prices, the degree of relative cheapness for CTD bonds in the cash market varies from 5 to 20 cents, and this price effect is highly significant. While the R^2 values in each of these regressions is small, the discrimination of a simple dummy variable regression involving 1,224 CTD noncallable bond observations out of the full sample of 24,380 observations is also small. Given the noisy estimates for synthetic prices and the numerous market frictions involved in the relative pricing of Treasury securities, the low R^2 values are not surprising (e.g, Elton and Green, 1998). Nonetheless, the economic significance of this simple regression deserves emphasis. Bonds that are CTD in the futures market are priced significantly different in the cash market than non-CTD bonds, with the implication that this is an additional factor in the relative pricing of Treasury securities. We explore in the next subsection whether this pricing factor is robust to the presence of alternate market frictions used as controls.

Looking at the second row of regression results for the three pricing methods in Table 2, the coefficient for the price slack variable is positive and significant in each of the three cases. The implication here, as in the case study examined in Jordan and Kuipers (1997), is that while CTD bonds are relatively cheap in general, they would be even *cheaper* in the absence of price pressure effects from the bond futures market. In particular, when the futures contract has a large amount of price slack, CTD bond prices are often bid up in the cash market by nearly the full amount of slack (the coefficient is equal to 0.93 and 0.86 for the first two estimation methods), which, referring to Table 1, amounts to a price increase of roughly 20 cents on average. Thus, bonds can become relatively rich in the cash market despite their inexpensive CTD status in the futures market. With the pricing errors constructed from linearly-interpolated yields (*DPLIY*), the coefficient on price slack is much lower in value at 0.16 (though still statistically significant); the smaller coefficient estimate likely reflects the overfitting of bond prices inherent to this estimation method.[x]

Overall, the average slack value from Table 1 and the average of the three estimated price adjustment coefficients for the regressions shown in Table 2, lead to an average estimated price pressure effect of 14 cents in the cash market for bonds that are CTD. For context, this price premium is larger than the bid-ask spread for all but the most illiquid of long-term Treasury bonds (Fleming, 2003). Given the enormous leverage used by institutional traders in this market, even modest price premia that exceed transactional costs can be exploited for profitable trading strategies (Fontaine and Garcia, 2012).

4.2. Regression tests for cash bond pricing: controlling factors.

To examine the robustness of the impact of CTD status in the bond futures market as a significant factor in the pricing of cash market U.S. Treasury bonds, we expand on the tests in Table 2 by including control variables previously shown to be significant in the analysis of systematic bond pricing errors. In particular, we estimate the following pooled time series and cross-sectional regression for the complete sample of 24,380 daily observations on noncallable U.S. Treasury bonds deliverable against the lead or first-deferred CBT bond futures contract over the period 1990–1994, as well as for the subsample of 1,224 noncallable CTD bond observations over the same sample period:

$$\begin{aligned}\left(P_{actual}^{cash} - P_{predicted}^{cash}\right)_{j,t} = \;& \beta_0 + \beta_1 \cdot CTD_{j,t} + \beta_2 \cdot CTD_{j,t} \cdot SLACK_{j,t} + \beta_3 \cdot ONRUN_{j,t} \\ & + \beta_4 \cdot AGE_{j,t} + \beta_5 \cdot S_{1,j,t} + \beta_6 \cdot S_{2,j,t} + \beta_7 \cdot PREMIUM_{j,t} \\ & + \beta_8 \cdot DISCOUNT_{j,t} + \varepsilon_{j,t} \quad . \end{aligned} \qquad (2)$$

The dependent variable in Eq. (2) is the same bond price prediction error examined in Eq. (1) using the three alternate estimation methods *DPSPL*, *DPCIR*, and *DPLIY*. Similarly, the dummy variable $CTD_{j,t}$ and price slack variable $SLACK_{j,t}$ are defined and constructed as before.

The remaining independent variables in Eq. (2) control for liquidity and tax effects in the relative pricing of long-term Treasury bonds. The dummy variable $ONRUN_{j,t}$ reflects the well-known result (e.g., Krishnamurthy, 2002) that the most-recently auctioned Treasury securities ("on-the-run" issues) are quoted at relatively higher prices than are off-the-run securities, reflecting their greater liquidity in the cash market. The variable $AGE_{j,t}$ controls for the finding in Sarig and Warga (1989) that over time, the liquidity of bonds is reduced as portfolio managers place the securities in static investment portfolios; hence, older bonds often trade at relatively lower prices. The dummy variables $S_{1,j,t}$ and $S_{2,j,t}$ control for the size of the quoted bid-ask spread for deliverable noncallable Treasury bonds in our sample. Since there are only three discrete spreads in the data ($1/16^{th}$, $1/8^{th}$, and $1/4^{th}$ of one point per \$100 face value bond), we follow Beim (1992) and dummy in the two larger spreads in Eq. (2), rather than use a continuous percentage spread variable; the smaller spread value of $1/16^{th}$ is captured in the regression intercept. The semi-continuous dummy variables $PREMIUM_{j,t}$ and $DISCOUNT_{j,t}$ account for the differential taxation of premium and discount bonds, which leads to the possibility of tax-timing options (Constantinides and Ingersoll, 1984) and tax-clientele effects (Ronn, 1987) as determinants of the relative pricing of long-term Treasury bonds. Finally, $\varepsilon_{j,t}$ in Eq. (2) is the regression residual; Table 3 reports the estimation results.[xi]

Table 3 Coefficient estimates from regressions of daily estimated pricing errors for deliverable, noncallable U.S. Treasury bonds on futures, tax, and liquidity-related variables, 1990–1994.

The dependent variable is the daily pricing error for 24,380 observations on noncallable U.S. Treasury bonds deliverable against the lead or first-deferred CBT Treasury bond futures contract over the period 1990–1994. The pricing error is calculated as the observed full (with accrued interest) price of the bond minus its synthetic price, where the synthetic price is estimated using fits to the daily U.S. Treasury yield curve from three methods: cubic splines (*DPSPL*), the one-factor Cox, Ingersoll and Ross (1985) term structure model (*DPCIR*), and linear interpolation (*DPLIY*). The pricing errors are measured in units of dollars assuming a \$100 face value bond. Heteroskedasticity and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987) are used in constructing the *t*-statistics (shown in parentheses). Results are reported separately for the full sample of observations on deliverable noncallable bonds (regression A) and for the subsample of observations involving only cheapest-to-deliver (CTD) noncallable bonds (regression B).

	(A) All observations (N=24,380)			(B) CTD observations (N=1,224)		
Indt. Variable	*DPSPL*	*DPCIR*	*DPLIY*	*DPSPL*	*DPCIR*	*DPLIY*
Intercept	0.1083	0.4319	–0.0605	–0.0530	0.0219	–0.0102
	-22.1	*-47.55*	*-14.5*	*-3.18*	*-0.76*	*-0.92*
CTD[a]	–0.2089	–0.2266	–0.0423			
	-21.38	*-16.53*	*-6.09*			
CTD•SLACK[b]	0.8906	0.8618	0.2351	0.8696	0.8582	0.2246
	-13.52	*-9.66*	*-5.2*	*-12.62*	*-12.45*	*-5.83*
ONRUN[c]	0.2619	0.8116	0.4455			
	-6.9	*-14.26*	*-15.77*			
AGE[d]	0.0241	–0.0381	0.0086	0.0099	–0.0122	–0.0011
	-19.76	*-18.39*	*-8.75*	*-4.26*	*-2.15*	*-0.55*
S_1[e]	0.0077	–0.1260	0.0462	–0.0173	–0.1069	–0.0373
	-1.23	*-13.33*	*-7.43*	*-2.72*	*-9.3*	*-4.36*
S_2[f]	0.0625	–0.0030	0.1276	–0.1043	–0.1098	–0.0452
	-2.46	*-0.11*	*-5.1*	*-2.97*	*-3.62*	*-1.58*
PREMIUM[g]	–0.0059	–0.0055	0.0002	–0.0031	–0.0026	–0.0014
	-23.06	*-13.91*	*-0.8*	*-8.21*	*-3.57*	*-2.9*
DISCOUNT[h]	0.0016	0.0016	0.0062	0.0038	0.0018	0.006
	-1.81	*-1.28*	*-7.67*	*-4.52*	*-1.3*	*-5.8*
adj. R^2	0.184	0.455	0.292	0.148	0.452	0.256

[a] Dummy variable *CTD* is equal to one if the bond observation is cheapest to deliver against the lead or first-deferred CBT Treasury bond futures contract (or both), else zero. The coefficient is not estimated in regression B as every observation in the subsample is a CTD bond.

[b] Price slack in the Treasury bond futures contract on day *t*, calculated as the allowable increase in the synthetic price of the CTD bond such that the bond would no longer be CTD, measured in units of dollars assuming a $100 face value bond (see Appendix B for details). For bond observations CTD against only the first-deferred contract, the slack for that contract is used in the regression analysis. For bond observations CTD against the lead or both contracts, the slack used is for the lead contract.

[c] Dummy variable *ONRUN* is equal to one if the bond observation is the most recently issued 30-year bond on day *t*, else zero. The coefficient is not estimated in regression B as no on-the-run bonds are the cheapest-to-deliver bond during the sample period.

[d] The age of the bond, in years, since the original issuance date.

[e] Dummy variable S_1 is equal to one if the quoted price spread for the bond on day *t* is equal to $0.125 (1/8th), else zero.

[f] Dummy variable S_2 is equal to one if the quoted price spread for the bond on day *t* is equal to $0.250 (1/4th), else zero.

[g] Continuous dummy variable *PREMIUM* is equal to the quoted percentage premium to par value for the bond on day *t*, if the flat (quote) price of the bond is greater than 100, else zero.

[h] Continuous dummy variable *DISCOUNT* is equal to the quoted percentage discount to par value for the bond on day *t*, if the flat (quote) price of the bond is less than 100, else zero.

Examining the results for the complete sample of observations in Table 3 (regression A), the coefficient estimates for the futures market variables *CTD* and *SLACK* are little changed from the results in Table 2. Thus, our earlier conclusion regarding the impact of a bond's status in the futures market on the relative prices of Treasury bonds in the cash market is robust to the presence of controlling factors for systematic bond pricing errors. Using any of the three estimation methods, the coefficient for the CTD dummy remains negative and highly significant, while the coefficient for the slack variable is positive and highly significant.

For the control variables in regression A, the coefficient estimates for on-the-run bonds and the size of the premium or discount to par value are generally as expected. The results show that on-the-run bonds are quoted, on average, at prices 26 to 81 cents higher than otherwise similar off-the-run bonds, depending on the estimation method used to generate bond price prediction errors. The smaller estimates are consistent with the values found in Jordan and Jordan (1997), while the larger on-the-run price premium estimates are consistent with those found in Carayannopolous (1996). Since only one bond is on-the-run at any given time, and it remains so until a new bond is issued, it is likely that estimates of the on-the-run price premium for bonds are highly sample-specific, which may account for the varied estimates across different studies. In the case of the coefficient estimates for the semi-continuous dummy variables *PREMIUM* and *DISCOUNT*, each is less than one cent for any of the dependent variables used in regression A. This result is consistent with the evidence in Green and Ødegaard (1997), who show that the economic significance of tax-timing options and tax-clientele effects in the U.S. Treasury market declined subsequent to the Tax Reform Act of 1986. Despite this, the sign of the coefficients for premium and discount bonds are as expected, in that the tax rules continue to favor discount bonds (at least for taxable investors) at the expense of premium bonds.[xii]

In the case of the liquidity variables *AGE*, S_1, and S_2 however, the results in regression A are somewhat surprising when using the prediction errors constructed from *DPSPL* and *DPLIY*. In particular, if older bonds are less liquid, the evidence in Sarig and Warga (1989) suggests that the coefficient for the age variable should be negative, reflecting a relatively lower price than for younger bonds. Similarly, if the larger price spreads S_1 and S_2 (compared to the $1/16^{th}$ spread captured by the intercept) indicate reduced liquidity, then the coefficients for these dummy variables should also be negative. Yet, the results for *DPSPL* and *DPLIY* in regression A of Table 3 show positive estimated coefficients that are highly significant in all but one instance for each of these three variables. A likely explanation for these results can be traced to the market practice of matrix pricing for illiquid securities (Fleming, 2003; Livingston, Wu and Zhou, 2019), a supposition addressed in the next regression.

Specifically, regression B in Table 3 presents results for the subsample of observations involving only CTD bonds. As shown, the general conclusions remain unchanged from the analysis contained in regression A.[xiii] The slack effect is positive and highly significant in each case, with the remaining variables mostly of the same sign and statistical significance as before. The primary exception to this concerns the spread variable dummies S_1 and S_2, which are now of the expected sign and statistically significant. Since cash market trading activity in CTD bonds would, as a rule, be expected to be greater than for non-CTD off-the-run bonds, the changed signs on the spread variables in regression B may simply reflect the deletion of the most illiquid bonds from the sample that are present in regression A; these would be the same bonds most likely subject to matrix pricing in practice (Fleming, 2003).

5. Summary and conclusions

This study addresses an issue of persistent concern to financial analysts and market regulators alike; the impact that derivatives markets can have on prices in the primary asset market. We find that bonds that are the cheapest-to-deliver (CTD) issue in the CBT Treasury futures market are priced at a relative premium in the primary cash Treasury market. On average, CTD bond prices are bid up by roughly 14 cents in value (per $100 face value bonds) compared to what we estimate their price would be in the absence of the bond's CTD status in the futures market. This price pressure effect is larger than typical transaction costs, at least for large traders, and is robust to the inclusion of control variables.

Our paper also adds to the growing literature concerning the idiosyncratic pricing of US Treasury market securities and their resulting richness/cheapness on a relative basis. A common thread throughout this literature seeks to identify issue characteristics and features that create heterogeneity among Treasury securities, and explore the valuation impact associated with such factors. A deliverable bond's CTD status in the futures market is one such widely followed factor, and this study shows that CTD bonds exhibit both economically and statistically significant price premiums in the cash Treasury market. One implication of our finding is that this observable feature of deliverable bonds should be used as an additional control variable in future studies exploring richness/cheapness within the US Treasury market.

Appendix A. Identifying the cheapest-to-deliver (CTD) bond

Let $P_{k,t}^{F}$ be the closing price of the *k*th outstanding Treasury bond futures contract on day *t*, and $X_{j,k}$ be the conversion factor for deliverable bond *j* against the same contract. The method for calculating conversion factors is well-known and can be found in any of a number of sources; they are also published by the CBT on a regular basis. The contract invoice price on day *t*, to be received by the short party for delivery of the *j*th bond into the *k*th futures contract on the delivery date, $P_{j,k,t}^{INV}$, is

$$P_{j,k,t}^{INV} = \left(P_{k,t}^{F} \cdot X_{j,k}\right) + I_{j,k} \quad , \tag{A.1}$$

where $I_{j,k}$ is the accrued interest for the bond on the chosen delivery date.[xiv] Further, let $P_{j,t}$ be the full (with accrued interest) price of the *j*th deliverable bond in the cash market on day *t* assuming next-day cash market delivery. Then the (annualized) money market return on investment, or "implied repo rate" ($IRR_{j,k,t}$), to a short party in the *k*th futures contract who buys bond *j* on day *t*, and eventually delivers it into the futures contract, is

$$IRR_{j,k,t} = \left(\frac{P_{j,k,t}^{INV} - P_{j,t}}{P_{j,t}}\right) \cdot \left(\frac{360}{N_{k,t}}\right) \quad , \tag{A.2}$$

where $N_{k,t}$ is the number of (calendar) days between day *t* and the eventual delivery date. Eq. (A.2) assumes that the bond purchaser is short $X_{j,k}$ contracts for every bond bought, and the calculation ignores any variation margin that occurs over the term $N_{k,t}$. If the bond in question makes a semiannual coupon payment, C_j, between day *t* and the delivery date, the implied repo rate is easily modified to

$$IRR_{j,k,t} = \left[\frac{P_{j,k,t}^{INV} - P_{j,t} + C_j}{P_{j,t} - \left(N_{k,t}^{*}/N_{k,t}\right) \cdot C_j}\right] \cdot \left(\frac{360}{N_{k,t}}\right) \quad , \tag{A.3}$$

where $N_{k,t}^{*}$ is the number of days between the coupon payment date and the eventual delivery date, and the received coupon is assumed to be reinvested at the same $IRR_{j,k,t}$. Using either of these two equations, the cheapest-to-deliver (CTD) bond for the *k*th contract on day *t* is the bond with the largest $IRR_{j,k,t}$.

Appendix B. Calculation of futures price slack

Let the cash price $P_{ctd,t}$ of the CTD bond for contract *k* on day *t* increase by an arbitrary amount $\Delta P_{ctd,t}$. Assume that the bond's basis,

$$Basis_{ctd,k,t} = P_{ctd,t} - \left(P_{k,t}^{F} \cdot X_{ctd,k}\right) \quad , \tag{B.1}$$

remains constant, which from Eq. (A.1) implies that the futures invoice price $P_{j,k,t}^{INV}$ also increases by $\Delta P_{ctd,t}$, while the futures price itself increases by ($\Delta P_{ctd,t}/X_{ctd,k}$). (Since a bond's basis in the futures market varies according to the aggregate value of the various quality options embedded in the contract, an impli-cation of this assumption is that these option values remain constant.) From Eq. (A.2) the new implied repo rate, $IRR_{ctd,k,t}^{*}$, for the CTD bond will decrease in magnitude and is related to the observed *IRR* via

$$IRR_{ctd,k,t}^{*} = IRR_{ctd,k,t} \cdot \left(\frac{P_{ctd,t}}{P_{ctd,t} + \Delta P_{ctd,t}}\right) \quad . \tag{B.2}$$

The extension of this equation to bonds that make intermediate coupon payments prior to the delivery date should be obvious from inspection of Eq. (A.3).

Conversely, assume that the price of the next-cheapest-to-deliver bond remains constant. Due to the increase in the futures invoice price by $\Delta P_{ctd,t}$, this implies from Eq. (A.2) that the new *IRR* for the next-cheapest-to-deliver bond will increase in magnitude and is related to its own observed *IRR* via

$$IRR_{j,k,t}^{*} = IRR_{j,k,t} + \left(\frac{\Delta P_{ctd,t}}{P_{j,t}}\right) \cdot \left(\frac{360}{N_{k,t}}\right) \quad . \tag{B.3}$$

As before, Eq. (B.3) must be modified if the next-cheapest bond pays an intermediate coupon.

At some point, the price increase $\Delta P_{ctd,t}$ will be large enough that the current CTD bond will no longer be CTD based on the above discussion. Thus, equating Eqs. (B.2) and (B.3) and solving for $\Delta P_{ctd,t}$,

$$\left(\Delta P_{ctd,t}\right)^2 + \left[P_{ctd,t} + \left(\frac{N_{k,t}}{360}\right) \cdot P_{j,t} \cdot IRR_{j,k,t}\right] \cdot \Delta P_{ctd,t} + \left[\left(\frac{N_{k,t}}{360}\right) \cdot P_{ctd,t} \cdot P_{j,t} \cdot \left(IRR_{j,k,t} - IRR_{ctd,k,t}\right)\right] = 0 \quad , \tag{B.4}$$

the positive root represents the maximum cash market price increase for the CTD bond such that a CTD substitution does not occur. This allowable price increase is the "slack" variable in the futures contract studied in Jordan and Kuipers (1997).

Finally, to construct the upper bound for the slack variable as used in this paper, the analysis of Eqs. (A.1) – (A.3) and (B.2) – (B.4) is repeated using a synthetic cash market price for the CTD bond. The synthetic price is chosen as the lower of the actual bond price or the price of a coupon STRIPS replicating portfolio for the bond (Jordan and Kuipers, 1997). If the price of the STRIPS replicating portfolio is in fact the lower of the two, the resulting slack measure will be greater than the observed slack as described above, because [from Eq. (A.2) or (A.3)] the initial $IRR_{ctd,k,t}$ will be even larger than before.

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i A partial list of studies in the "richness/cheapness" literature that examine the impact of market frictions on the relative pricing of Treasury securities includes: Garbade and Silber (1976), Constantinides and Ingersoll (1984), Litzenberger and Rolfo (1984a), Ronn (1987), Cornell and Shapiro (1989), Sarig and Warga (1989), Amihud and Mendelson (1991), Beim (1992), Kamara (1994), Jordan and Jordan (1997), Green and Ødegaard (1997), Elton and Green (1998), Jordan, Jorgensen and Kuipers (2000), Fleming (2002), Krishnamurthy (2002), Longstaff (2004), Vayanos and Weill (2008), and Fleckenstein, Longstaff and Lustig (2014).

ii For CBT Treasury futures, there is little trading activity in contracts of horizon longer than the first-deferred on any given day.

iii During our sample period, a notional coupon of 8% was used by the CBT in its conversion factor calculation. The notional coupon was lowered to 6% beginning with trading in the March 2000 contract to reflect the lower interest rate environment in effect at that time.

iv Yield levels below the notional coupon tend to make short-duration bonds cheap to deliver after accounting for the conversion factor system, while positive yield curve slope tends to favor the delivery of long-duration bonds. This conversion factor bias is well-known and discussed in, for example, Kane and Marcus (1987), Livingston (1987) and elsewhere.

v Because all deliverable Treasury bonds and all Treasury STRIPS (during our sample period) make payments on synchronized calendar dates, a replicating portfolio of STRIPS can be constructed for any deliverable bond. The approach here closely follows that in Carayannapolous (1995) and Jordan and Kuipers (2005).

vi We track the lead futures contract until the first calendar day of the contract expiration month, at which time we switch into the next contract. By doing so, we avoid potential contamination problems in our slack calculation associated with volatility in the value of the contract's quality option during the delivery month (Chance and Hemler, 1993). Our switching approach to the next contract is commonly used in bond futures research.

vii The data used in this study are often highly autocorrelated and, in places, consist of pooled cross-sections and time series. Consequently, all *t*-statistics reported in the paper, including the regression results in Section 4, utilize heteroskedasticity and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987). In general, these HAC standard errors are substantially larger than their univariate or ordinarly least squares counterparts, so the reported *t*-statistics throughout the paper can be regarded as conservative.

viii If two different noncallable bonds are CTD against the lead and deferred contracts on a given day, and in addition, these bonds are of adjacent available maturities, we exclude the one from the other in calculating a linearly interpolated yield. Otherwise, the calculation follows Jegadeesh (1993). For the longest maturity noncallable bond on a given day (the on-the-run 30-year bond), there will not be a longer maturity observation available to bracket the bond's maturity; in this case, we linearly extrapolate a synthetic yield using the two closest maturity bonds (assuming neither is itself CTD).

ix While our data constraints for this paper are in part driven by a desire to measure CTD effects over similar sample periods as those studied in extant research, our primary constraint is admittedly acquiring the necessary coincident data sets on a complete and daily basis, from three different market sectors. We are grateful to both Jordan and Kuipers (1997) and Jordan and Kuipers (2005) for making their raw data available for this study.

x Jegadeesh (1993) notes that, using linear interpolation to generate synthetic bond yields, average yield errors are less than one basis point with corresponding price errors of less than one cent (per $100 face value). This degree of overfitting can often obscure any meaningful price discrimination that may exist in the market data.

xi We do not control for several additional market frictions or bond characteristics previously found to be significant determinants of systematic bond pricing errors because they are not applicable to the current study given our limited scope of analyzing only long-term noncallable Treasury bonds deliverable in the bond futures market. For example, while Amihud and Mendelson (1991) explore the impact of bond maturity on the relative price differences between Treasury securities, this variable is superfluous in the present study since we examine only original issue 30-year bonds and already control for the bond's age. Similarly, the distinction between bills, notes, and bonds explored in Kamara (1994) is not relevant to this study. In the case of repo market specials as studied in Jordan and Jordan (1997), we do not have access to this data for the complete sample period. However, in an examination of every coupon-bearing U.S.

Treasury security on special in the repo market over the subperiod September 1991–December 1992, using daily data, Jordan and Jordan (1997) report that only four observations involve long-term noncallable Treasury bonds; during this time period at least, repo market specials occur exclusively for shorter maturity Treasury notes. Finally, while we do not control for the daily trading volume in Treasury bonds as in Elton and Green (1998) due to data availability, we do control for bond age, which they show to be highly correlated with volume.

[xii] We classify the premium or discount to par value for bonds based on their flat (net of accrued interest) price; this is the correct classification for tax purposes. Nonetheless, the conclusions from regression A are not changed if we alter the definition based on the bond's full price, or if we ignore the size of the premium or discount and use indicator variables as an alternative. In all cases, the coefficient estimates are not economically significant.

[xiii] On-the-run bonds are never CTD during our sample period, so the coefficient for *ONRUN* is not estimated in Regression B.

[xiv] Since delivery can be made at any time during the contract expiration month, the determination of the CTD bond is conditional on the delivery date chosen due to timing differences in the cash flow payment schedule for deliverable bonds. However, when the yield curve is positively sloped, it will almost always be optimal to make delivery on the last possible day. This is because the short futures position is (assumed) long in the cash instrument for calculating CTD (a position which will generate positive carry), and the short holds all the embedded options in the futures contract including physical delivery rights. Thus, we make the common assumption that the delivery date is the last business day of the contract expiration month in all of our CTD calculations. During our sample period, the overwhelming majority of actual bond deliveries against the futures contract do in fact occur on the last possible day, which we confirmed in the CBT's market summary reports and delivery records.