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MULTIPLE CONTRACTS: PERIODIC BALLOON PAYMENTS AND CONSTANT AMORTIZATION

Clovis de Faro[1], Gerson Lachtermacher[2]

[1]*Graduate School of Economics, EPGE-FGV, Rio de Janeiro, Brazil*
[2]*Associate Professor FCE-UERJ, Rio de Janeiro, Brazil*

Abstract

De-Losso, et al. (2013) should be credited as the first to show that, considering its cost of capital, a financial institution may be better off if a single contract is substituted by multiple contracts with their analysis focusing attention only in the case of constant installments. De Faro (2022) expanded the analysis to the case where the constant amortization scheme of debt amortization is the one considered by the financial institution. In this paper the case of balloon payments will be added to the constant amortization multiple contracts scheme.

Keywords

Multiple Contracts, Ballon payment, Constant Amortization Method

1. Introduction

As far as it is known, at least in Brazil, the idea of substituting a single contract by multiple individual contracts, was first presented in Sandrini (2007), later in Vieira Sobrinho (2012), both addressing the concept of anatocism (the charge of interest upon interest).

Nevertheless, the pioneering work of De-Losso, et al. (2013) should be credited as the first to show that, considering its cost of capital, a financial institution may be better off if a single contract is substituted by multiple contracts with their analysis focusing attention only in the case of constant installments.

De Faro (2022) expanded the analysis to the case where the constant amortization scheme of debt amortization is the one considered by the financial institution. That is, in a similar vein, it was shown that if a loan of F units of capital, with a term of n periods, at the periodic rate of interest i of compound interest, the financial institution providing the loan will be better off if a single contract is substituted by n individual contracts with the value of the k^{th} one being the present value of the k^{th} payment of the single contract.

The same type of results are also observed, considering compound interest, if the system of amortization is of periodic payments of interest only, as in de Faro (2021), or if the financial institution makes use of two distinct versions of the so called SACRE system of amortization, as in de Faro and Lachtermacher (2023a and 2023b), and also in the case of the German system of amortization, as in de Faro and Lachtermacher (2024).

However, all the above studies addressed only cases wherein no periodic balloon payments were present. One exception is presented in de Faro and Lachtermacher (2025) which took into consideration the so called “décimo terceiro salário” (thirteenth salary), which is very relevant in the formal labor sector in Brazil.

Taking into account that, particularly in the case of the Brazilian System of Housing Financing ("Sistema Financeiro de Habitação"), the borrower is entitled to choose either the system of constant installments (which, in Brazil, is known as "Tabela Price," honoring the classical work of Richard Price (1771)), or the system of constant amortization (usually denoted, in Brazil, as "SAC"). We are going to focus attention on this system of amortization which, in Italy, is known as "ammortamento italiano"; cf. Marcelli (2019).

2. The Case of a Single Contract

In the classical case where there are no balloon payments, the adoption of the system of constant amortization implies that the sequence of the n periodic payments, with the k^{th} one denoted as p_k for $k=1,2,\ldots,n$, follows an arithmetic progression with ratio $R=-i\times F/n$, and first payment $p_1=F\times(i+1/n)$; cf. de Faro and Lachtermacher (2012).

Suppose now that besides the n periodic payments, the borrower also must pay ℓ balloon payments with periodicity m, such that $n=\ell\times m$.

Denoting by p'_j the j^{th} balloon payment, for $j=1,2,\ldots,\ell$, if the first one has to be paid at the end of m periods, it follows that we must have:

$$F=\sum_{k=1}^{n} p_k\times(1+i)^{-k}+\sum_{j=1}^{\ell} p'_j\times(1+i_m)^{-j} \quad (1)$$

and

$$i_m=(1+i)^m-1 \quad (2)$$

denotes the interest rate relative to m periods.

For our purposes it is convenient to suppose that the full amount F is split into two sub-loans, the first denoted F_1 and the second F_2, such that $F=F_1+F_2$ with F_1 to be repaid in accordance with the classical system of constant amortization, so that:

$$F_1=\sum_{k=1}^{n} P'_k\times(1+i)^{-k} \quad (3)$$

with the sequence of payments following an arithmetic progression with ratio $R'=-i\times F_1/n$, and with first term $P'_1=F_1\times(i+1/n)$.

Following, we are going to consider four cases for the second part of the loan, F_2. The first will be termed as pure constant amortization, the second as constant balloon payments, the third as extra amortization and the fourth as thirteen amortization per year, as illustrated in the four numerical examples that will be considered.

2.1 Pure Constant Amortization

In this case, F_2 will also be paid by the constant amortization method. Denoting by A''_k the k^{th} parcel of amortization, and assuming ℓ parcels with value equal to F_2/ℓ, with periodicity m, and $n=\ell\times m$, we will have:

$$A''_k=\begin{cases}0,\ \text{for } k=1,2,\ldots,m-1,m+1,\ldots,2m-1,2m+1,\ldots,\ell\times m-1\\ F_2/\ell,\ \text{for } k=m,2m,\ldots,\ell\times m\end{cases} \quad (4)$$

with the parcel of interest of the installment, P''_k, being denoted by I''_k, the outstanding debt at time k being denoted by S''_k for $k=1,2,\ldots,n$, with $S''_0=F_2$ and $S''_n=0$. Also, $A''_k=P''_k-I''_k$, $I''_k=i\times S''_{k-1}$ and $S''_k=S''_{k-1}-A''_k$.

To illustrate this case of pure constant amortization, consider loan $F=\$100,000.00$, at the monthly compounded interest rate $i=1\%$, with term $n=12$ months, with, $\ell=4$ and $m=3$.

Employing the artifice of considering two different loans, with $F_1 = \$70,000.00$ and $F_2 = 30,000.00$, Tables 1 and 2 show the corresponding evolutions of the two debts.

In the first debt, F_1, the sequence of the 12 periodic payments follows an arithmetic progression with ratio equal to $-\$58.33$, and first payment $P_1' = \$6,533.33.$ In Table 1, noting that the parcels of amortization are all equal to \$5,833.33 $(A_k' = 70000/12)$ with the outstanding debt being $S_k' = S_{k-1}' - A_k'$, with $S_0' = \$70,000.00,$ and with the k^{th} parcel of interest being $I_k' = i \times S_{k-1}'$, for $k = 1, 2, \ldots, 12$, we have the evolution of the debt F_1.

Table 1. Evolution of the first debt, F_1

k	I_k'	A_k'	P_k'	S_k'
0	-	-	-	70.000,00
1	700.00	5,833.33	6,533.33	64,166.67
2	641.67	5,833.33	6,475.00	58,333.33
3	583.33	5,833.33	6,416.66	52,500.00
4	525.00	5,833.33	6,358.33	46,666.67
5	466.67	5,833.33	6,300.00	40,833.33
6	408.33	5,833.33	6,241.67	35,000.00
7	350.00	5,833.33	6,183.33	29,166.67
8	291.67	5,833.33	6,125.00	23,333.33
9	233.33	5,833.33	6,066.67	17,500.00
10	175.00	5,833.33	6,008.33	11,666.67
11	116.67	5,833.33	5,950.00	5,833.33
12	58.33	5,833.33	5,891.67	0,00
$\sum$	4,550.00	70,000.00	74,550.00	-

Table 2 shows the evolution of the debt F_2. Here the first balloon payment, considering its parcel of interest which occurs at time 3, is equal to \$7,800.00. It should be noted that whenever there is no amortization, as for instance in epochs 1 and 2, there is only the payment of interest.

Table 2. Evolution of the second debt, F_2 in the case of pure constant amortization

k	I_k''	A_k''	P_k''	S_k''
0	-	-	-	30,000,00
1	300.00	0.00	300.00	30,000.00
2	300.00	0.00	300.00	30,000.00
3	300.00	7,500.00	7,800.00	22,500.00
4	225.00	0.00	225.00	22,500.00
5	225.00	0.00	225.00	22,500.00
6	225.00	7,500.00	7,725.00	15,000.00
7	150.00	0.00	150.00	15,000.00
8	150.00	0.00	150.00	15,000.00
9	150.00	7,500.00	7,650.00	7,500.00
10	75.00	0.00	75.00	7,500.00
11	75.00	0.00	75.00	7,500.00
12	75.00	7,500.00	7,575.00	0,00
$\sum$	2,250.00	30,000.00	32,250.00	-

Consolidating the results in Tables 1 and 2, Table 3 presents the evolution of the considered full loan $F = \$100,000.00.$

Table 3. Evolution of full debt of the loan in the case of pure constant amortization

k	$I_k = I'_k + I''_k$	$A_k = A'_k + A''_k$	$P_k = P'_k + P''_k$	$S_k = S'_k + S''_k$
0	-	-	-	100,000.00
1	1,000.00	5,833.33	6,833.33	94,166.67
2	941.67	5,833.33	6,775.00	88,333.33
3	883.33	13,333.33	14,216.67	75,000.00
4	750.00	5,833.33	6,583.33	69,166.67
5	691.67	5,833.33	6,525.00	63,333.33
6	633.33	13,333.33	13,966.67	50,000.00
7	500.00	5,833.33	6,333.33	44,166.67
8	441.67	5,833.33	6,275.00	38,333.33
9	383.33	13,333.33	13,716.67	25,000.00
10	250.00	5,833.33	6,083.33	19,166.67
11	191.67	5,833.33	6,025.00	13,333.33
12	133.33	13,333.33	13,466.67	0,00
$\sum$	6,800.00	100,000.00	106,800.00	-

It should be noted that $F = \sum_{k=1}^{n} P_k \times (1+i)^{-k}$.

2.2 The Case of Constant Balloon Payments

Another option to consider is one where the periodic balloon payments remain constant. In this case, denoting by P^* the value of the ℓ constant periodic balloon payments, we will have, cf. de Faro and Lachtermacher (2012):

$$P^* = F_2 \times i_m / \left\{1-\left(1+i_m\right)^{-\ell}\right\} \qquad (5)$$

Considering the same numerical example of the previous section, we will have $P^* = \$8,076.62$, $\hat{P}''_j = P^*$, for $j = 1,2,\ldots,\ell$, $\hat{S}''_0 = \$30,000.00$, the k^{th} parcel of interest being $\hat{I}''_k = i \times \hat{S}''_{k-1}$, and $\hat{S}''_k = \hat{S}''_{k-1} - \hat{A}''_k$, for $k = 1,2,\ldots,12$.

It should be observed that the loan is taken at the compound interest rate of 1% per month. Therefore, in the first two periods, since there is no payment, we must consider that the debt is increased, which implies that we have negative parcels of amortization. The same happens in epochs 4,5,7,8,10 and 11. Table 4 presents the corresponding evolution of debt F_2.

Table 4. Evolution of the debt F_2 in the case of constant balloon payments

k	$\hat{I}''_k$	$\hat{A}''_k$	$\hat{P}''_k$	$\hat{S}''_k$
0	-	-	-	30,000.00
1	300.00	-300.00	0.00	30,300.00
2	303.00	-303.00	0.00	30,603.00
3	306.03	7,770.59	8,076.62	22,832.41
4	228.32	-228.32	0.00	23,060.73
5	230.61	-230.61	0.00	23,291.34
6	232.91	7,843.71	8,076.62	15,447.63
7	154.48	-154.48	0.00	15,602.11
8	156.02	-156.02	0.00	15,758.13
9	157.28	7,919.04	8,076.62	7,839.09
10	78.39	-78.39	0.00	7,917.48
11	79.17	-79.17	0.00	7,996.65
12	79.97	7,996.65	8,076.62	0.00
$\sum$	2,306.18	30,000.00	32,306.49	-

Therefore, considering the results already presented in Table 1, which refers to the debt F_1, it follows that the consolidation of Tables 1 and 4 implies that the evolution of full debt, in this case of constant balloon payments, is as depicted in Table 5.

Table 5. Evolution of full debt in the case of constant balloon payments

k	$I_k = I'_k + \hat{I}''_k$	$A_k = A'_k + \hat{A}''_k$	$P_k = P'_k + \hat{P}''_k$	$S_k = S'_k + \hat{S}''_k$
0	-	-	-	100,000.00
1	1,000.00	5,533.33	6,533.33	94,466.67
2	944.67	5,530.33	6,475.00	88,936.33
3	889.36	13,603.92	14,493.28	75,332.41
4	753.32	5,605.05	6,358.33	69,727.40
5	697.28	5,602.72	6,300.00	64,124.67
6	641.24	13,677.04	14,318.29	50,447.63
7	504.48	5,678.85	6,183.33	44,768.78
8	447.69	5,677.31	6,125.00	39,091.46
9	390.61	13,752.37	14,143.29	25,339.09
10	253.39	5,754.94	6,008.33	19,584.41
11	195.84	5,754.16	5,950.00	13,829.98
12	138.30	13,829.98	13,968.29	0.00
∑	6,856.18	100,000.00	106,856.48	-

2.3 Extra Amortization

This case differs from the first case because extra amortizations are not necessarily constant. They are interpreted as if they are the result of extraordinary parcels of amortization evenly spaced.

That is, considering the full amount F of the loan, denoting as B_k the extra amortization at epoch k and $F_2 = \sum_{j=1}^{\ell} B_j$, $F_1 = F - F_2$ and the hypothesis of constant amortization method, the k^{th} parcel of amortization of F_2 will be:

$$\tilde{A}''_k = \begin{cases} 0, \text{ for } k = 1, 2, \ldots, m-1, m+1, \ldots, 2m-1, 2m+1, \ldots, n-1 \\ B_1, \text{ for } k = m, B_2, \text{ for } k = 2m, \ldots B_\ell, \text{ for } k = \ell \times m \end{cases} \quad (6)$$

So considering $B_3 = 6000,\ B_6 = 7000,\ B_9 = 8000$ and $B_{12} = 9000$, if $F_2 = \tilde{S}''_0 = \$30,000.00,$ the k^{th} parcel of interest will be $\tilde{I}''_k = i \times \tilde{S}''_{k-1},$ with $\tilde{P}''_k = \tilde{A}''_k + \tilde{I}''_k$ and $\tilde{S}''_k = \tilde{S}''_{k-1} - \tilde{A}''_k,$ for $k = 1, 2, \ldots, 12$. Table 6 presents the corresponding evolution of debt F_2.

Table 6. Evolution of the debt F_2 in the case of extra amortization

k	$\tilde{I}''_k$	$\tilde{A}''_k$	$\tilde{P}''_k$	$\tilde{S}''_k$
0				30,000.00
1	300.00	0.00	300.00	30,000.00
2	300.00	0.00	300.00	30,000.00
3	300.00	6,000.00	6,300.00	24,000.00
4	240.00	0.00	240.00	24,000.00
5	240.00	0.00	240.00	24,000.00
6	240.00	7,000.00	7,240.00	17,000.00
7	170.00	0.00	170.00	17,000.00
8	170.00	0.00	170.00	17,000.00
9	170.00	8,000.00	8,170.00	9,000.00
10	90.00	0.00	90.00	9,000.00
11	90.00	0.00	90.00	9,000.00
12	90.00	9,000.00	9,090.00	0.00
∑	2,400.00	30,000.00	32,400.00	

It should be noted that, in contrast to the previous case, we do not have negative parcels of amortization since now, for instance in epochs 1 and 2, the parcels of interest are included in the payments.

Consequently, considering the results already presented in Table 1, which refers to the debt F_1, it follows that the consolidation of Tables 1 and 6 implies that the evolution of the full debt, in this case of extra amortization, is as depicted in Table 7.

Table 7. Evolution of full debt in the case of extra amortization

k	$I_k = I'_k + \tilde{I}''_k$	$A_k = A'_k + \tilde{A}''_k$	$P_k = P'_k + \tilde{P}''_k$	$S_k = S'_k + \tilde{S}''_k$
0			-	100,000.00
1	1,000.00	5,833.33	6,833.33	94,166.67
2	941.67	5,833.33	6,775.00	88,333.33
3	883.33	11,833.33	6,716.67	76,500.00
4	765.00	5,833.33	6,598.33	70,666.67
5	706.67	5,833.33	6,540.00	64,833.33
6	648.33	12,833.33	6,481.67	52,000.00
7	520.00	5,833.33	6,353.33	46,166.67
8	461.67	5,833.33	6,295.00	40,333.33
9	403.33	13,833.33	6,236.67	26,500.00
10	265.00	5,833.33	6,098.33	20,666.67
11	206.67	5,833.33	6,040.00	14,833.33
12	148.33	14,833.33	5,981.67	0.00
$\sum$	6,950.00	100,000.00	106,950.00	-

2.4 Thirteen Amortizations Per Year

On July 13, 1962 it was established, by law, that employees in the formal sector of the Brazilian economy would be entitled to receive thirteen monthly wages per year. This resulted in the so-called "décimo terceiro salário" (thirteenth wage), also known as "gratificação de Natal" (Christmas gratification).

To consider this peculiarity, we will analyze the case where the loan *F* has a term of ℓ years. That is, considering that up to now the term *n* of the loan has been measured in number of periods, in this case, months, we are going to suppose that the number of years of the contract is $\ell = n/12$ years. Additionally, we are going to assume that the periodic rate of interest *i* is a monthly rate as in the other cases.

We should be pointed out that we have a particular instance of the Extra Amortizations case where only one extra amortization is done per year with the same value as the regular one. Furthermore, it should also be pointed out that the annual extra amortization is usually made every December when the thirteenth salary is usually paid.

To account for this peculiarity, we will assume the case of a contract that was signed at the end of December, with the periodicity *m* of the balloon payments being fixed to 12 months so that the first extra amortization occurs in December of the following year.

Furthermore, in this very particular situation, the partition of *F* is not arbitrary. We must have $F_2 = F \times \ell / (n + \ell)$ with $F_1 = F - F_2$.

With this proviso, given that we are considering the system of constant amortization, denoting by A_k^{13} the parcel of amortization in epoch *k* of the loan F_2, we will have:

$$A_k^{13} = \begin{cases} 0, \text{ for } k = 1, 2, \ldots, 11, 13, \ldots, 23, 25, \ldots, 12\ell - 1 \\ F / (\ell + n), \text{ for } k = 12, 24, \ldots, 12\ell = n \end{cases} \quad (7)$$

As a numerical example, consider the case where $F = \$100,000.00$, $i = 1\%$ per month, and the term *n* of the loan is equal to 24 months.

Noting that we will have $F_2 = S_0^{13} = \$7,692.31$, *m*=12, ℓ=2, *n*=24, the *k*th parcel of interest being $I_k^{13} = i \times S_k^{13}$, with $P_k^{13} = A_k^{13} + I_k^{13}$ and $S_k^{13} = S_{k-1}^{13} - A_k^{13}$, for $k = 1, 2, \ldots, 24$, Table 8 presents the corresponding evolution of debt F_2.

Table 8. Evolution of the debt F_2 – Thirteen Amortizations Per Year

k	I_k^{13}	A_k^{13}	$\tilde{P}_k''$	$\tilde{S}_k''$
0				7,692.31
1	76.92	0.00	76.92	7,692.31
2	76.92	0.00	76.92	7,692.31
:	:	:	:	:
11	76.92	0.00	76.92	7,692.31
12	76.92	3,846.15	3,923.07	3,846.15
13	38.46	0.00	38.46	3,846.15
:	:	:	:	:
23	38.46	0.00	38.46	3,846.15
24	38.46	3,846.15	3,884.62	0.00
∑	1,384,62	7,692.31	9,076.93	

In this case, the first part of the loan, $F_{1,}$ is equal to $92,307.69 and the evolution of its debt using the constant amortization method is presented in Table 9.

Table 9. Evolution of the first debt, F_1, in the case of Thirteen Amortizations Per Year

k	I_k'	A_k'	P_k'	S_k'
0	-	-	-	92,307.69
1	923.08	3,846.15	4,769.23	88,461.53
2	884.62	3,846.15	4,730.77	84,615.38
:	:	:	:	:
11	538.46	3.846.15	4.384,62	50,000.00
12	500.00	3.846.15	4.346,15	46,153.84
13	461.54	3.846.15	4.307,69	42,307.69
:	:	:	:	:
23	76.92	3,846.15	3,923.08	3,846.15
24	38.46	3,846.15	3,884.62	0.00
∑	11,538.46	92,307.69	103,846.15	

Consolidating Tables 8 and 9, Table 10 presents the evolution of the full debt F.

Table 10. Evolution of full debt in the case of Thirteen Amortizations Per Year

k	$I_k = I_k' + I_k^{13}$	$A_k = A_k' + A_k^{13}$	$P_k = P_k' + P_k^{13}$	$S_k = S_k' + S_k^{13}$
0	-	-	-	100,000.00
1	1,000.00	3,846.15	4,846.15	96,153.85
2	961.54	3,846.15	4,807.69	92,307.69
3	923.08	3,846.15	4,769.23	88,461.54
4	884.62	3,846.15	4,730.77	84,615.38
5	846.15	3,846.15	4,692.31	80,769.23
6	807.69	3,846.15	4,653.85	76,923.08
7	769.23	3,846.15	4,615.38	73,076.92
8	730.77	3,846.15	4,576.92	69,230.77
9	692.31	3,846.15	4,538.46	65,384.62
10	653.85	3,846.15	4,500.00	61,538.46
11	615.38	3,846.15	4,461.54	57,692.31
12	576.92	7,692.31	8,269.23	50,000.00
13	500.00	3,846.15	4,346.15	46,153.85
14	461.54	3,846.15	4,307.69	42,307.69
15	423.08	3,846.15	4,269.23	38,461.54
16	384.62	3,846.15	4,230.77	34,615.38
17	346.15	3,846.15	4,192.31	30,769.23
18	307.69	3,846.15	4,153.85	26,923.08
19	269.23	3,846.15	4,115.38	23,076.92

20	230.77	3,846.15	4,076.92	19,230.77
21	192.31	3,846.15	4,038.46	15,384.62
22	153.85	3,846.15	4,000.00	11,538.46
23	115.38	3,846.15	3,961.54	7,692.31
24	76.92	7,692.31	7,769.23	0.00
∑	12,923.08	100,000.00	112,923.08	-

It is interesting to note that the effect of having thirteen amortizations per year is to decrease the constant amortization by 7.69%, as given by $100\times\left\{1-n/\left(n+\ell\right)\right\}$. It should be also noted that some small divergences in tables 8,9 and 10 are the result of the more precise value of the amortization which is \$3,846.153846.

3. The Case of Multiple Contracts

Instead of a single contract, the financial institution providing the loan has the option of requiring the borrower to sign *n* individual contracts, one for each of the *n* payments that would be associated with the case of a single contract, with the value of the loan of the k^{th} subcontract being the present value at the same interest rate *i* of the corresponding payment of the single contract. That is, the value of the loan of the k^{th} subcontract, denoted by F_k , is:

$$F_k = \frac{P_k}{\left(1+i\right)^k} \quad ,k=1,2,...,n \tag{8}$$

where P_k, denotes the k^{th} payment of the corresponding single contract. It should be noted thar $P_k = P_k'$ so that the payment made by the borrower at the k^{th} epoch is the same in both cases.

In the case of multiple contracts, the parcel of amortization associated with the k^{th} payment, denoted as A_k' will be:

$$A_k' = F_k = \frac{P_k}{\left(1+i\right)^k} \quad ,k=1,2,...,n \quad . \tag{9}$$

Namely, the parcel of amortization associated with the k^{th} subcontract is exactly equal to the value of the loan of the k^{th} subcontract.

On the other hand, from an accounting point of view, it follows that the parcel of interest associated with the k^{th} subcontract, which will be denoted by I_k' . Hence,

$$I_k' = P_k \times \left[1-\frac{1}{\left(1+i\right)^k}\right] = P_k - A_k' \quad ,k=1,2,...,n \quad . \tag{10}$$

It should be noted that although from the strict accounting point of view, not taking into consideration the costs that may be associated with the bookkeeping and registration of the *n* subcontracts, the total of interest payments is the same in both cases. That is:

$$\sum_{k=1}^{n} I_k = \sum_{k=1}^{n} I_k' \tag{11}$$

where I_k denotes the k^{th} installment of interest of the corresponding single contract.

However, in terms of present values, and depending on the financial institution's cost of capital, it is possible that the financial institution will be better off adopting the multiple contracts option, as will be illustrated considering the cases of single contracts that have been analyzed.

3.1 The case of the Pure Constant Amortization

We will begin the analysis with the case of the pure constant amortization version of a single contract presented in Table 3. Table 11 presents the values of the sequence of payments P_k of the sequence of the parcels of interest I_k, in the case of the corresponding single contract, and the sequence I_k' of the parcels of interest in the case of the adoption of the option of multiple contracts.

Additionally, Table 11 presents also the sequence of differences, d_k, and the sequence of accumulated values of d_k, denoted as Δ_k , respectively given by:

$$d_k = I_k - I'_k \quad , k = 1, 2, ..., n \tag{12}$$

and

$$\Delta_k = \sum_{\ell=1}^{k} d_\ell \tag{13}$$

Table 11. Multiple Contracts – Pure Constant Amortization

k	$F_k = A'_k$	I'_k	$P'_k = P_k$	I_k	$d_k = I_k - I'_k$	Δ_k
1	6,765.68	67.66	6,833.33	1,000.00	932.34	932.34
2	6,641.51	133.49	6,775.00	941.67	808.17	1.740.52
3	13,798.56	418.11	14,216.67	883.33	465.22	2.205.74
4	6,326.45	256.88	6,583.33	750.00	493.12	2.698.86
5	6,208.31	316.69	6,525.00	691.67	374.98	3.073.84
6	13,157.23	809.43	13,966.67	633.33	-176.10	2.897.74
7	5,907.21	426.12	6,333.33	500.00	73.88	2.971.62
8	5,794.86	480.14	6,275.00	441.67	-38.48	2.933.14
9	12,541.69	1,174.97	13,716.67	383.33	-791.64	2.141.50
10	5,507.16	576.17	6,083.33	250.00	-326.17	1.815.33
11	5,400.35	624.65	6,025.00	191.67	-432.98	1.382.35
12	11,950.98	1,515.68	13,466.67	133.33	-1,382.35	0.00
∑	100,000.00	6,800.00	106,800.00	6,800.00	0.00	

It is interesting to note that the sequence of the values of d_k has more than one change of sign. However, adopting the proposition in Nordstrom (1972), as the sequence of accumulated values of d_k does not change sign, it follows that d_k has a unique internal rate of return, which, in this case, is zero.

As previously noted, the first point that should be observed is that although the sum of the parcels of interest is the same in the case of a single contract and in the case of multiple contracts, the timing and the values of their respective payments are not the same. Consequently, a more comprehensive comparison should consider the cost of capital of the financial institution providing the loan.

Denoting by ρ the financial institution's cost of capital, with ρ being relative to the same period as the financing rate i, we define as $V_s(\rho)$ and $V_m(\rho)$ the present values of the interest sequences of the single and multiple contracts. Hence,

$$V_s(\rho) = \sum_{k=1}^{n} I_k \times (1+\rho)^{-k} \tag{14}$$

and

$$V_m(\rho) = \sum_{k=1}^{n} I'_k \times (1+\rho)^{-k} \quad . \tag{15}$$

Table 12 presents the present values of the corresponding interest sequences for several values of its cost of capital for the case of our numerical example, with ρ_a denoting the cost of capital in annual terms.

Table 12. Present values of interest sequences. – Pure Amortization Case

ρ_a	ρ	$V_s(\rho)$	$V_m(\rho)$	%(difference)
5%	0.40741%	6,669.69	6,571.56	1.49337
10%	0.79741%	6,548.67	6,361.78	2.93772
15%	1.17149%	6,435.88	6,168.38	4.33661
20%	1.53095%	6,330.43	5,989.44	5.69321
25%	1.87693%	6,231.54	5,823.31	7.01035
30%	2.21045%	6,138.57	5,668.61	8.29057

Therefore, in the case of our simple numerical example, we have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$. That is, the financial institution providing the loan should prefer to implement the multiple contracts option. Furthermore, considering different values of the financing rate *i*, as well as distinct values of the term *n* of the contract, it can be shown that we always have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$.

3.2 The case of the Constant Balloon Payments

For the case of constant balloon payments, Table 13 presents the corresponding values of the multiple contracts, using the same notation as section 3.1.

Table 13. Multiple Contracts – Constant Balloon Payments

k	$F_k = A'_k$	I'_k	$P'_k = P_k$	I_k	$d_k = I_k - I'_k$	Δ_k
1	6,468.65	64.69	6,533.33	1,000.00	935.31	935.31
2	6,347.42	127.58	6,475.00	944.67	817.08	1,752.40
3	14,067.04	426.25	14,493.29	889.36	463.12	2,215.51
4	6,110.23	248.10	6,358.33	753.32	505.22	2,720.74
5	5,994.23	305.77	6,300.00	697.27	391.51	3,112.25
6	13,488.47	829.81	14,318.29	641.25	-188.57	2,923.68
7	5,767.31	416.03	6,183.33	504.48	88.45	3,012.13
8	5,656.33	468.67	6,125.00	447.69	- 20.98	2,991.15
9	12,931.77	1,211.52	14,143.29	390.91	- 820.60	2,170.55
10	5,439.27	569.07	6,008.33	253.39	- 315.68	1,854.87
11	5,333.13	616.87	5,950.00	195.84	- 421.03	1,433.84
12	12,396.15	1,572.14	13,968.29	138.30	- 1.433.84	0.00
$\sum$	100,000.00	6,856.49	106,856.49	6,856.49	0.00	

Also in this case, the sequence of the values of d_k has more than one change of sign. However, as the sequence of accumulated values of d_k does not change sign, it follows that d_k has a unique internal rate of return, which, in this case, is zero.

As previously noted, although the sum of the parcels of interest is the same in both cases, the timing and the values of their respective payments are not the same. Consequently, a more comprehensive comparison should consider the cost of capital of the financial institution providing the loan.

Table 14 presents the present values of the corresponding interest sequences for several values of its cost of capital, using the same notation as section 3.1.

Table 14. Present values of interest sequences. – Constant Balloon Payments Case

ρ_a	ρ	$V_s(\rho)$	$V_m(\rho)$	%(difference)
5%	0.40741%	6,724.58	6,625.15	1.50086
10%	0.79741%	6,602.08	6,412.74	2.95268
15%	1.17149%	6,487.93	6,216.93	4.35902
20%	1.53095%	6,381.20	6,035.77	5.72304
25%	1.87693%	6,281.13	5,867.61	7.04755
30%	2.21045%	6,187.06	5,711.04	8.33511

Therefore, in the case of our simple numerical example, we have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$. That is, the financial institution providing the loan should prefer to implement the multiple contracts option. Furthermore, considering different values of the financing rate *i*, as well as distinct values of the term *n* of the contract, it can be shown that we always have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$.

3.3 The case of the Extra Amortization Case

For the case of Extra Amortization, Table 15 presents the corresponding values of the multiple contracts, using the same notation as in section 3.1.

Table 15. Multiple Contracts – Extra Amortization Case

k	$F_k = A'_k$	I'_k	$P'_k = P_k$	I_k	$d_k = I_k - I'_k$	Δ_k
1	6,765.68	67.66	6,833.33	1,000.00	932.34	932.34
2	6,641.51	133.49	6,775.00	941.67	808.17	1,740.52
3	12,342.67	374.00	12,716.67	883.33	509.34	2,249.85
4	6,340.87	257.46	6,598.33	765.00	507.54	2,757.39
5	6,222.59	317.41	6,540.00	706.67	389.25	3,146.64
6	12,700.34	781.33	13,481.67	648.33	-132.99	3,013.65
7	5,925.87	427.46	6,353.33	520.00	92.54	3,106.18
8	5,813.33	481.67	6,295.00	461.67	- 20.01	3,086.18
9	13,017.15	1,219.52	14,236.67	403.33	- 816.18	2,269.99
10	5,520.74	577.59	6,098.33	265.00	- 312.59	1,957.40
11	5,413.80	626.20	6,040.00	206.67	- 419.54	1,537.86
12	13,295.47	1,686.20	14,981.67	148.33	- 1.537.86	0.00
∑	100,000.00	6,950.00	106,950.00	6,950.00	0.00	

Also in this case, the sequence of the values of d_k has more than one change of sign. However, as the sequence of accumulated values of d_k does not change sign, it follows that d_k has a unique internal rate of return, which, in this case, is zero.

As previously noted, although the sum of the parcels of interest is the same in both cases, the timing and the values of their respective payments are not the same. Consequently, a more comprehensive comparison should consider the cost of capital of the financial institution providing the loan.
Table 16 depicts the present values of the corresponding interest sequences for several values of its cost of capital, using the same notation as section 3.1.

Table 16. Present values of interest sequences. – Extra Amortization Case

ρ_a	ρ	$V_s(\rho)$	$V_m(\rho)$	%(difference)
5%	0.40741%	6,814.90	6,712.81	1.52079
10%	0.79741%	6,689.47	6,495.10	2.99246
15%	1.17149%	6,572.59	6,294.47	4.41855
20%	1.53095%	6,463.35	6,108.90	5.80223
25%	1.87693%	6,360.95	5,936.69	7.14630
30%	2.21045%	6,264.69	5,776.39	8.45329

Therefore, in the case of our simple numerical example, we have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$. That is, the financial institution providing the loan should prefer to implement the multiple contracts option. Furthermore, considering different values of the financing rate *i*, as well as distinct values of the term *n* of the contract, it can be shown that we always have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$.

3.4 The case of Thirteen Amortizations Per Year

Considering our simple numerical example, as shown in Table 10, Table 17 presents the consolidated evolution of the debt in the case of multiple contracts, using the same notation as in section 3.1.

Table 17. Multiple contracts – Thirteen Amortizations Per Year

k	$F_k = A'_k$	I'_k	$P'_k = P_k$	I_k	$d_k = I_k - I'_k$	Δ_k
1	4,798.17	47.98	4,846.15	1,000.00	952.02	952.02
2	4,712.96	94.73	4,807.69	961.54	866.81	1,818.83
3	4,628.97	140.26	4,769.23	923.08	782.81	2,601.64
4	4,546.18	184.59	4,730.77	884.62	700.02	3,301.66
5	4,464.57	227.74	4,692.31	846.15	618.42	3,920.08
6	4,384.13	269.71	4,653.85	807.69	537.98	4,458.06
7	4,304.85	310.53	4,615.38	769.23	458.70	4,916.76
8	4,226.71	350.21	4,576.92	730.77	380.56	5,297.32
9	4,149.70	388.77	4,538.46	692.31	303.54	5,600.86
10	4,073.79	426.21	4,500.00	653.85	227.64	5,828.50
11	3,998.98	462.56	4,461.54	615.38	152.83	5,981.32
12	7,338.52	930.71	8,269.23	576.92	- 353.79	5,627.54
13	3,818.80	527.35	4,346.15	500.00	- 27.35	5,600.19
14	3,747.53	560.16	4,307.69	461.54	- 98.62	5,501.57
15	3,677.30	591.93	4,269.23	423.08	- 168.85	5,332.71
16	3,608.09	622.68	4,230.77	384.62	- 238.06	5,094.65
17	3,539.89	652.42	4,192.31	346.15	- 306.26	4,788.39
18	3,472.69	681.16	4,153.85	307.69	- 373.47	4,414.92
19	3,406.47	708.92	4,115.38	269.23	- 439.69	3,975.23
20	3,341.22	735.70	4,076.92	230.77	- 504.93	3,470.30
21	3,276.93	761.53	4,038.46	192.31	- 569.22	2,901.07
22	3,213.58	786.42	4,000.00	153.85	- 632.57	2,268.51
23	3,151.17	810.37	3,961.54	115.38	- 694.98	1,573.52
24	6,118.78	1,650.45	7,769.23	76.92	- 1,573.52	0.00
$\sum$	100,000.00	12,923.08	112,923.08	12,923.08	0.00	

In this case, the sequence of the values of d_k has only one change of sign. Therefore, not being necessary to consider the sequence of accumulated values of d_k, we can conclude that d_k has a unique internal rate of return, which, also in this case, is null.

As previously noted, although the sum of the parcels of interest is the same in both cases, the timing and the values of their respective payments are not the same. Consequently, a more comprehensive comparison should consider the cost of capital of the financial institution providing the loan.
Table 18 shows the present values of the corresponding interest sequences for several values of its capital cost, using the same notation as section 3.1.

Table 18. Present values of interest sequences. – Thirteen Amortizations Per Year Case

ρ_a	ρ	$V_s(\rho)$	$V_m(\rho)$	%(difference)
5%	0.40741%	12,469.23	12,101.34	3.04005
10%	0.79741%	12,057.70	11,372.90	6.02135
15%	1.17149%	11,682.70	10,723.36	8.94627
20%	1.53095%	11,339.45	10,141.07	11.81705
25%	1.87693%	11,023.97	9,616.52	14.63574
30%	2.21045%	10,732.93	9,141.85	17.40428

Therefore, in the case of our simple numerical example, we have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$. That is, the financial institution providing the loan should prefer to implement the multiple contracts option. Furthermore, considering different values of the financing rate *i*, as well as distinct values of the term *n* of the contract, it can be shown that we always have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$.

4. General Analysis

Taking into consideration that in all the cases that have been studied it was found that the option of multiple contracts is the better choice, it remains to provide numerical evidence of what can be defined as fiscal gain, which will be denoted by δ and given by the following expression, in percentage:

$$\delta(\%) = 100 \times \left\{ V_s(\rho) / V_m(\rho) - 1 \right\} \quad (16)$$

Initially, in order to contrast with the case of no balloon payments, which was considered in de Faro (2022), Tables 19 to 22 present the values of the fiscal gain δ (%) when the financing interest rate *i* varies from 0.5% monthly to 2% monthly, and the term of the contract goes from 5 to 30 years, for a loan of 100.000 units of capital, with the annual value of the financial institution cost of capital ranging from 5% to 30%.

Table 19. Fiscal Gains δ (%) – Constant Amortization – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.6825	15.5168	23.4761	31.5349	39.6694	47.8572
10	14.8685	30.7903	47.5872	65.0710	83.0556	101.3670
15	21.3873	45.0337	70.3895	96.8771	123.9663	151.2181
20	27.2393	57.9093	90.8068	124.7845	158.9431	192.6764
25	32.4511	69.2677	108.3712	148.0569	187.2419	225.3663
30	37.0662	79.1121	123.1035	166.9707	209.6860	250.8725

Table 20. Fiscal Gains δ (%) – Constant Amortization – *i*=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.1227	14.3365	21.6170	28.9421	36.2911	43.6457
10	12.8714	26.3602	40.3073	54.5600	68.9799	83.4467
15	17.3961	35.9156	55.1307	74.6588	94.1925	113.5060
20	20.9617	43.3693	66.4382	89.5578	112.3234	134.5030
25	23.7884	49.1477	74.9330	100.3992	125.1589	149.0551
30	26.0491	53.6374	81.3221	108.3384	134.3905	159.4140

Table 21. Fiscal Gains δ (%) – Constant Amortization – *i*=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.1916	12.3941	18.5893	24.7613	30.8961	36.9818
10	10.0435	20.2786	30.5991	40.9144	51.1509	61.2514
15	12.4877	25.2493	38.0549	50.7312	63.1598	75.2674
20	14.1168	28.4988	42.7979	56.7938	70.3689	83.4735
25	15.2546	30.7123	45.9355	60.7001	74.9199	88.5814
30	16.0821	32.2791	48.0986	63.3425	77.9656	91.9846

As it is shown in all the above tables, the values of δ are very significant and always positive, indicating that the best option for the financial institution is to implement multiple contracts.

As a further illustration, fixing $i = 1\%$ per month, Figures 1 and 2 depict the behavior of δ when the opportunity cost of the financial institution varies from to 5% to 30% in annual terms, and the term of the contract on annual terms varies from 5 to 30 years.

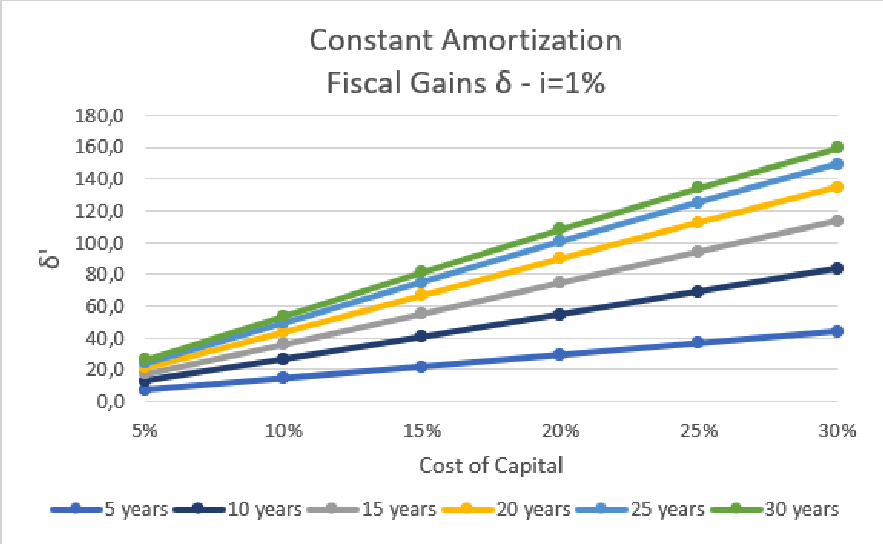

Figure 1. Constant Amortization Method-Fiscal Gain δ, when *i*=1%p.m.–Cost of Capital

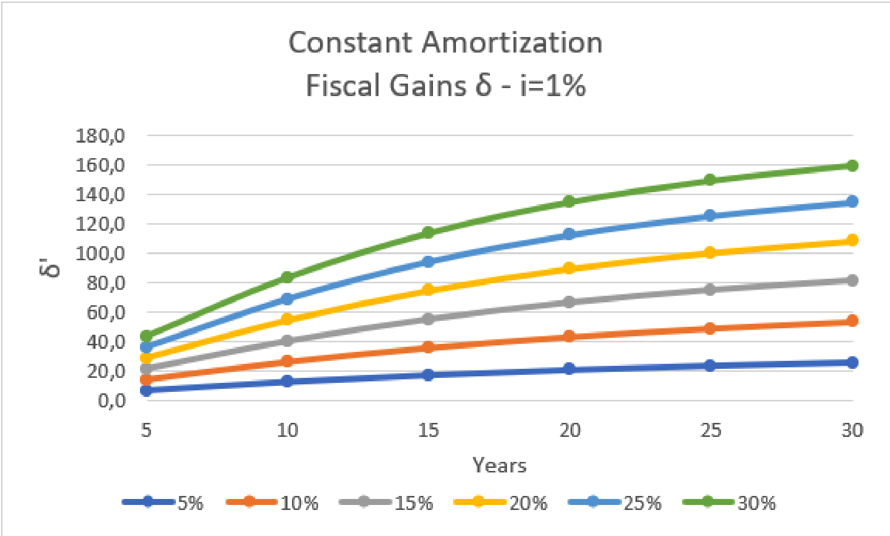

Figure 2. Constant Amortization Method - Fiscal Gain δ, when *i*=1%p.m. – Loan Term

4.1 The Case of the Pure Constant Amortization

Now, for the example of section 3.1, we generalize the comparison in the same way as in section 4 for the traditional constant amortization method. Tables 23 to 25 present the values of the fiscal gain δ (%) when the financing interest rate *i* varies from 0.5% monthly to 2% monthly, and the term of the contract goes from 5 to 30 years for a loan of 100.000 units of capital, with the annual value of the financial institution cost of capital ranging from 5% to 30%.

Table 23. Fiscal Gains δ (%) – Pure Constant Amortization – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.9159	16.0065	24.2432	32.5990	41.0484	49.5676
10	15.0708	31.2326	48.3024	66.0867	84.3940	103.0458
15	21.5666	45.4334	71.0397	97.7973	125.1680	152.7081
20	27.3983	58.2641	91.3770	125.5768	159.9591	193.9182
25	32.5916	69.5777	108.8582	148.7192	188.0790	226.3831
30	37.1901	79.3797	123.5136	167.5189	210.3748	251.7113

Table 24. Fiscal Gains δ (%) – Pure Constant Amortization – *i*=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.3238	14.7552	22.2683	29.8392	37.4461	45.0689
10	13.0222	26.6822	40.8165	55.2685	69.8961	84.5764
15	17.5128	36.1655	55.5228	75.1968	94.8773	114.3377
20	21.0534	43.5630	66.7362	89.9583	112.8248	135.1058
25	23.8613	49.2985	75.1597	100.6988	125.5312	149.5028
30	26.1076	53.7559	81.4965	108.5669	134.6750	159.7591

Table 25. Fiscal Gains δ (%) – Pure Constant Amortization – i=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.3438	12.7075	19.0716	25.4188	31.7341	38.0048
10	10.1339	20.4662	30.8884	41.3077	51.6494	61.8553
15	12.5459	25.3692	38.2372	50.9751	63.4643	75.6321
20	14.1566	28.5794	42.9183	56.9527	70.5658	83.7090
25	15.2831	30.7690	46.0190	60.8095	75.0558	88.7452
30	16.1032	32.3206	48.1591	63.4219	78.0651	92.1062

As it is shown in all the above tables, the values of δ are very significant and always positive, indicating that the best option for the financial institution is to implement multiple contracts. It is worth noting that the results observed are almost equal to those in the case of no balloon payments.

4.2 The Case of the Constant Balloon Payments

Similarly, considering the example of section 3.2, Tables 26 to 28 present the corresponding values of the fiscal gain.

Table 26. Fiscal Gains δ (%) – Constant Balloon Payments – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	8.0116	16.2135	24.5771	33.0749	41.6809	50.3707
10	15.3721	31.9397	49.5216	67.9209	86.9386	106.3856
15	22.1770	46.9483	73.7472	101.9498	130.9649	160.2913
20	28.3957	60.8251	96.0004	132.6218	169.6336	206.3070
25	34.0252	73.3140	115.5426	158,6877	201.4228	243.0533
30	39.0798	84.3032	132.1357	180,0270	226.6907	271.6650

Table 27. Fiscal Gains δ (%) – Constant Balloon Payments – *i*=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.4934	15.1194	22.8513	30.6640	38.5344	46.4413
10	13.4910	27.7530	42.6154	57.9090	73.4755	89.1745
15	18.3395	38.1170	58.8521	80.0928	101.4625	122.6773
20	22.2279	46.3669	71.4825	96.8071	121.8172	146.2098
25	25.3324	52.7927	80.9469	108.8207	135.9085	162.0166
30	27.8061	57.7244	87.8826	117.2747	145.5401	172.6296

Table 28. Fiscal Gains δ (%) – Constant Balloon Payments – *i*=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.6114	13.2752	19.9694	26.6745	33.3728	40.0491
10	10.7100	21.7360	32.9505	44.2404	55.5104	66.6841
15	13.3429	27.1455	41.1131	55.0131	68.6822	82.0177
20	15.0646	30.5922	46.1145	61.3274	76.0708	90.2826
25	16.2221	32.8172	49.1852	65.0251	80.2374	94.8216
30	17.0263	34.2890	51.1169	67.2679	82.7161	97.5074

As it is shown in all the above tables, the values of δ are very significant and always positive, indicating that the best option for the financial institution is to implement multiple contracts. Moreover, the fiscal gains are a little bigger than those observed in the traditional constant amortization method and in the pure constant amortization case.

4.3 The Case of Thirteen Amortizations Per Year

Similarly, considering the example of section 3.4, Tables 29 to 31 present the corresponding values of the fiscal gain. It should be noted that we did not make the analysis for the example of section 3.3 since this case is a special case of the Extra Amortization.

Table 29. Fiscal Gains δ (%) – Thirteen Amortizations Per Year – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.7445	15.6465	23.6787	31.8152	40.0318	48.3057
10	14.9213	30.9054	47.7729	65.3342	83.4019	101.8007
15	21.4338	45.1371	70.5574	97.1144	124.2759	151.6017
20	27.2804	58.0008	90.9537	124.9884	159.2045	192.9957
25	32.4874	69.3475	108.4965	148.2272	187.4571	225.6276
30	37.0981	79.1809	123.2090	167.1116	209.8630	251.0880

Table 30. Fiscal Gains δ (%) – Thirteen Amortizations Per Year – *i*=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.1762	14.4474	21.7892	29.1786	36.5949	44.0194
10	12.9107	26.4440	40.4396	54.7438	69.2173	83.7390
15	17.4263	35.9803	55.2320	74.7977	94.3692	113.7204
20	20.9854	43.4193	66.5151	89.6610	112.4525	134.6582
25	23.8072	49.1866	74.9914	100.4764	125.2547	149.1703
30	26.0641	53.6679	81.3669	108.3972	134.4636	159.5028

Table 31. Fiscal Gains δ (%) – Thirteen Amortizations Per Year – *i*=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.2320	12.4772	18.7169	24.9348	31.1169	37.2509
10	10.0671	20.3275	30.6743	41.0166	51.2803	61.4080
15	12.5028	25.2804	38.1021	50.7943	63.2384	75.3616
20	14.1271	28.5196	42.8289	56.8348	70.4197	83.5342
25	15.2620	30.7269	45.9570	60.7283	74.9549	88.6235
30	16.0875	32.2898	48.1142	63.3629	77.9912	92.0159

As it is shown in all the above tables, the values of δ are very significant and always positive, indicating that the best option for the financial institution is to implement multiple contracts. The fiscal gains, in this case, are almost equal to the traditional constant amortization method and the pure constant amortization.

5. The Case of Thirteen Constant Installments Per Year

Given that, in de Faro and Lachtermacher (2025), the case of thirteen constant payments per year was already considered, this section will focus on the comparison of the results associated with these two distinct systems of amortization: constant payments versus constant amortization, for the case of thirteen annual wages.

From de Faro and Lachtermacher (2025), where the system of constant payments was considered, a comparison was made of a single contract and the corresponding multiple one, using a loan of 100,000 units of capital.

Denoting by ρ the financial institution's cost of capital, with ρ being relative to the same period as the financing rate *i*, we define $V'_s(\rho)$ and $V'_m(\rho)$ as the present values of the interest sequences of the single ($\hat{I}_k$) and multiple ($\hat{I}'_k$)contracts, for the case of thirteen annual wages. Therefore,

$$V'_s(\rho) = \sum_{k=1}^{n} \hat{I}_k \times (1+\rho)^{-k} \quad (17)$$

And

$$V'_m(\rho) = \sum_{k=1}^{n} \hat{I}'_k \times (1+\rho)^{-k} \quad . \quad (18)$$

Tables 32 to 34 present a comparison of single and multiple contracts in terms of the values of the corresponding fiscal gains, here denoted by δ'.

Table 32. Fiscal Gains δ' (%) – Thirteen Constant Installments Per Year – *i*=0.5%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.9499	16.0955	24.4096	32.8657	41.4385	50.1043
10	15.7020	32.7705	51.0423	70.3276	90.4246	111.1317
15	23.0986	49.3869	78.3498	109.3411	141.6869	174.7680
20	30.0583	65.4209	104.8252	146.7593	189.8330	232.9811
25	36.5212	80.4147	129.1682	180.1247	231.2740	281.4229
30	42.4484	94.0214	150.5109	208.1736	264.8009	319.4594

Table 33. Fiscal Gains δ' (%) – Thirteen Constant Installments Per Year – i=1.0%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.5300	15.2162	23.0327	30.9551	38.9602	47.0264
10	14.0668	29.1469	45.0793	61.6900	78.8037	96.2536
15	19.5431	41.1647	64.3755	88.6429	113.4669	138.4274
20	24.0268	51.0542	80.0123	109.8471	139.7251	169.0926
25	27.6326	58.8604	91.8460	125.0606	157.5658	188.9256
30	30.4934	64.8031	100.2929	135.2076	168.7842	200.8451

Table 34. Fiscal Gains δ' (%) – Thirteen Constant Installments Per Year – *i*=2.0%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.7608	13.6135	20.5362	27.5082	34.5103	41.5252
10	11.3747	23.2906	35.6110	48.2020	60.9392	73.7132
15	14.3567	29.5902	45.3434	61.2890	77.1622	92.7714
20	16.2563	33.5050	51.1106	68.5772	85.5934	102.0036
25	17.4777	35.8855	54.3342	72.2754	89.4773	105.8967
30	18.2813	37.3235	56.0666	74.0310	91.1220	107.3949

As shown in all cases, the fiscal gain is substantial, which implies that the financial institution providing the loan should always choose the option of implementing multiple contracts for the case of thirteen constant installments per year.

5.1 Comparison of Thirteen Payments Per Year Methods

Given the financing institution could either offer the constant installments or the constant amortization methods, and since both multiple contracts versions offer significant fiscal gains over the corresponding single contracts, the financial institution should decide which method gives the best result, considering the corresponding interest sequences and its own cost of capital.

If the loan values are the same, and the cost of capital is not considered, the method which offers the bigger total interest payments should be chosen. Table 35 and Figure 3 present the total difference of interest charged by the method of constant installments and the method of constant amortization, both using thirteen wages per year.

Table 35. Total Difference of Interest - Thirteen Payments Per Year

	Total Interest Difference		
Term | *Interest Rate*	0.5	1.0	2.0
60	778.51	3,098.40	12,170.98
120	3,042.42	11,957.71	44,891.27
240	11,842.47	44,436.07	146,285.10
360	25,829.24	90,916.72	264,614.92

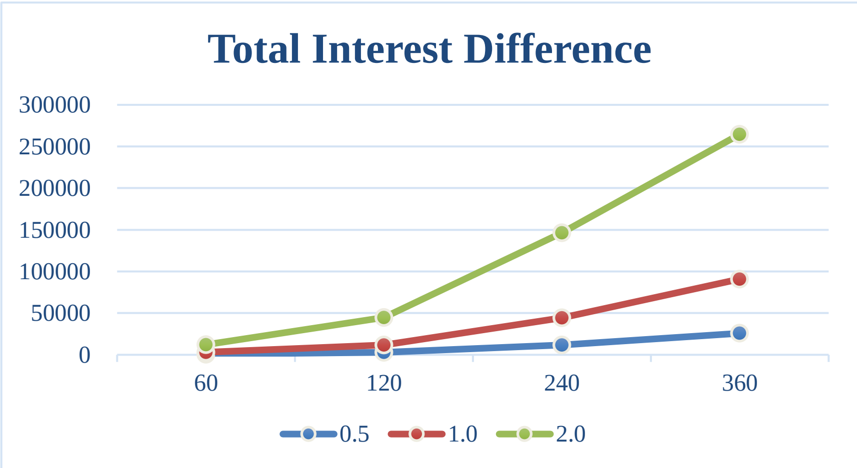

Figure 3. Total Interest Difference - Thirteen Payments Per Year

As can be shown, the total interest difference increases exponentially with the term and with the interest rate of the loan. On the other hand, considering the financial institution's cost of capital, Figures 4, 5 and 6 show the percentage difference of the present value of the corresponding interest sequences of both methods, for the case of thirteen wages per year.

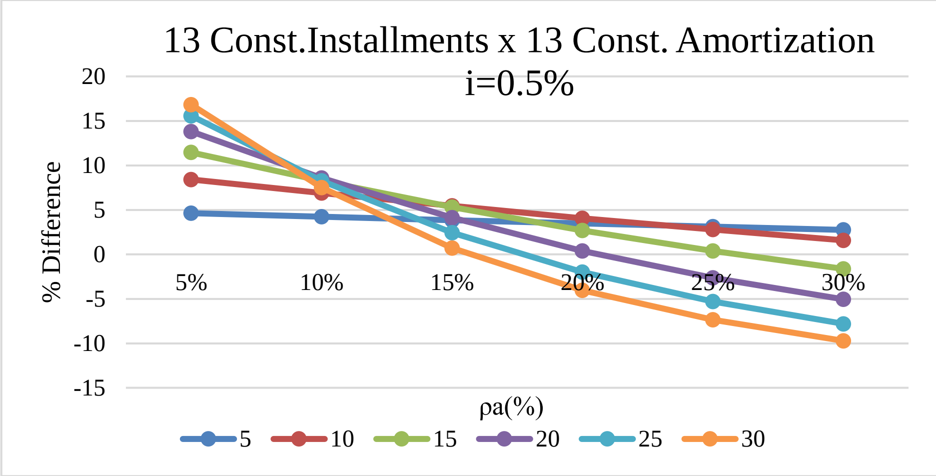

Figure 4. Percentage Difference of the Present Value – *i*=0.5% p.m.

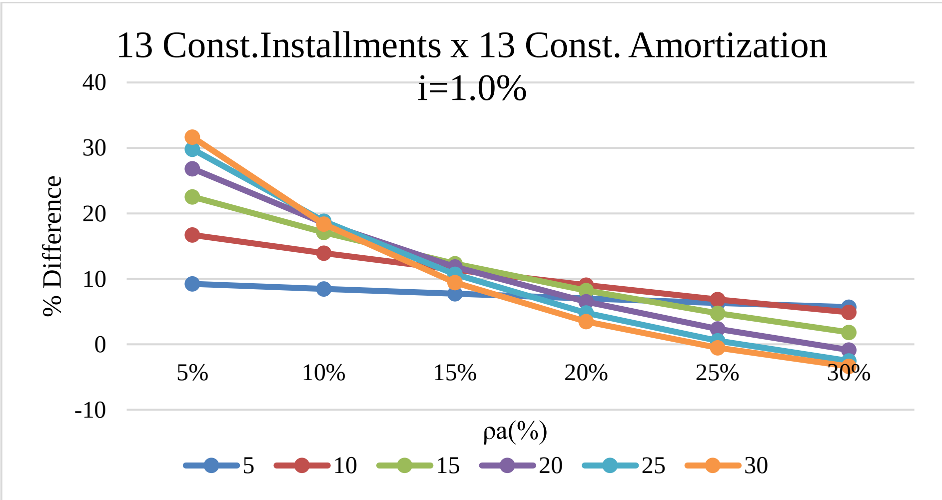

Figure 5. Percentage Difference of the Present Value – *i*=1.0 % p.m.

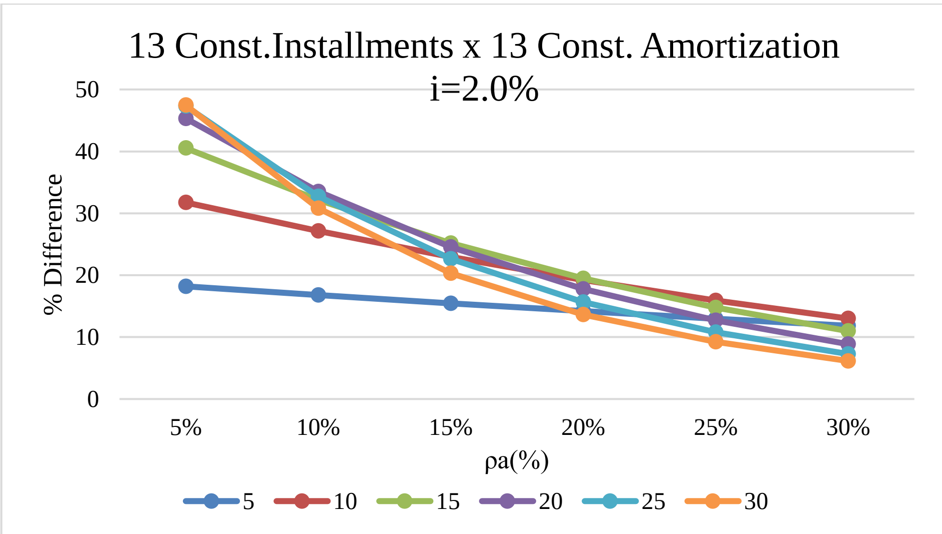

Figure 6. Percentage Difference of the Present Value – *i*=2.0% p.m.

As shown in Figures 4, 5 and 6, most of the present values of the constant installments interest sequences are larger than the constant amortization ones. However, increasing the cost of capital and the term of the loan, especially for low interest rates, we will have cases with bigger present values for the constant amortization method.

Therefore, if the criteria for choosing the best option method is the present value of the interest sequence, the choice will depend on the interest rate, on the term of the loan, as well as on the cost of capital of the financial institution.

6. Conclusions

Analogously to the case of the system of amortization with constant installments and constant balloon payments, it was shown that substituting a single contract by multiple contracts is always the best option for the financing institution when considering the system of constant amortization.

However, given the possibility that the financing institution providing the loan may have the option of requiring the borrower to implement either the constant installment method of amortization, or the constant amortization system, the best option depends on the interest rate being charged, on the term of the loan, and on the financial institution cost of capital.

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